Analyzing Ellipses: Foci, Axes, and Standard Form

Precalculus / Analytic Geometry Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Find the center, vertices, and foci of an ellipse given key information
- Determine the values of a, b, and c using the relationship $c^2 = a^2 - b^2$ and the major axis length
- Write the standard equation of an ellipse in the form $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$

Problems

1. Two foci of an ellipse are located at (1, 3) and (7, 3). Find the center of the ellipse.

2. The major axis of an ellipse has a length of 10. Find the value of a.

$$2a = 10$$

3. An ellipse has foci at (0, 2) and (6, 2). Find the value of c, which is the distance from the center to each focus.

4. An ellipse has a = 5 and c = 3. Use the relationship between a, b, and c to find b squared.

$$c^2 = a^2 - b^2$$

5. An ellipse has foci at (0, 1) and (4, 1) and a major axis of length 6. Find the values of a, b, and c.

6. Using the information from Problem 5, write the standard equation of the ellipse with center (2, 1), a squared = 9 (horizontal), and b squared = 5.

$$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{5} = 1$$

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7. An ellipse has foci at (2, 0) and (2, 4) and a major axis of length 8. Find the center, c, a, and b squared.

8. Write the standard equation of the ellipse from Problem 7 with center (2, 2), vertical major axis, a squared = 16, and b squared = 12.

$$\frac{(x-2)^2}{12} + \frac{(y-2)^2}{16} = 1$$

9. Given the equation of an ellipse, identify the center, the values of a and b, and determine whether the major axis is horizontal or vertical.

$$\frac{(x+3)^2}{25} + \frac{(y-4)^2}{9} = 1$$

10. An ellipse has foci at (–1, 2) and (5, 2) and passes through the point (2, 2 + sqrt(5)). Find c, a, b squared, and write the standard equation.

Analyzing Ellipses: Foci, Axes, and Standard Form — Answer Key

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Answer Key

1. Answer: (4, 3)

- The center is the midpoint of the two foci.
- x-coordinate of center: $(1 + 7) / 2 = 4$
- y-coordinate of center: $(3 + 3) / 2 = 3$
- Center = (4, 3)

2. Answer: $a = 5$

- The relationship between the major axis and a is: $2a$ = major axis length
- $2a = 10$
- $a = 10 / 2 = 5$

3. Answer: $c = 3$

- First, find the center: midpoint of (0, 2) and (6, 2) = (3, 2)
- The distance from the center (3, 2) to either focus is c.
- $c = |6 – 3| = 3$

4. Answer: $b^2 = 16$

- Use the formula: $c^2 = a^2 – b^2$
- Substitute: $3^2 = 5^2 – b^2$
- $9 = 25 – b^2$
- $b^2 = 25 – 9 = 16$

5. Answer: $a = 3$, $c = 2$, $b^2 = 5$

- Center = midpoint of (0,1) and (4,1) = (2, 1)
- c = distance from center to focus = $|4 – 2| = 2$
- $2a$ = major axis = 6, so $a = 3$
- $c^2 = a^2 – b^2 \to 4 = 9 – b^2 \to b^2 = 5$

6. Answer: $(x–2)^2/9 + (y–1)^2/5 = 1$

- Standard form: $(x–h)^2/a^2 + (y–k)^2/b^2 = 1$ for a horizontal major axis
- $h = 2$, $k = 1$, $a^2 = 9$, $b^2 = 5$
- Substitute to get: $(x–2)^2/9 + (y–1)^2/5 = 1$

7. Answer: Center = (2, 2), $c = 2$, $a = 4$, $b^2 = 12$

- Center = midpoint of (2, 0) and (2, 4) = (2, 2)
- Since foci share the same x-coordinate, the major axis is vertical
- c = distance from center (2,2) to focus (2,4) = 2

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- $2a = 8$, so $a = 4$
- $c^2 = a^2 - b^2 \rightarrow 4 = 16 - b^2 \rightarrow b^2 = 12$

8. Answer: $(x-2)^2/12 + (y-2)^2/16 = 1$

- For a vertical major axis, the larger denominator (a^2) goes under the y term
- $h = 2$, $k = 2$, $a^2 = 16$ (under y), $b^2 = 12$ (under x)
- Standard form: $(x-2)^2/12 + (y-2)^2/16 = 1$

9. Answer: Center (–3, 4), a = 5, b = 3, horizontal major axis

- Rewrite: $(x - (-3))^2/25 + (y - 4)^2/9 = 1$
- Center = $(h, k) = (-3, 4)$
- $a^2 = 25 \rightarrow a = 5$; $b^2 = 9 \rightarrow b = 3$
- Since 25 > 9 and 25 is under the x term, the major axis is horizontal

10. Answer: $c = 3$, $a^2 = b^2 + 9$; $b^2 = 5$, $a^2 = 14$; equation: $(x-2)^2/14 + (y-2)^2/5 = 1$

- Center = midpoint of (–1, 2) and (5, 2) = (2, 2)
- c = distance from center to focus = $|5 - 2| = 3$
- The point $(2, 2 + \sqrt{5})$ is on the ellipse; substitute into standard form: $(2-2)^2/a^2 + (2+\sqrt{5}-2)^2/b^2 = 1$
- This gives: $0 + 5/b^2 = 1$, so $b^2 = 5$
- Use $c^2 = a^2 - b^2$: $9 = a^2 - 5$, so $a^2 = 14$
- Standard equation: $(x-2)^2/14 + (y-2)^2/5 = 1$

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