Analyzing Ellipses: Standard Form, Vertices & Foci

Precalculus / Analytic Geometry Worksheet · Grade 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Identify the center, vertices, and orientation of an ellipse from its graph or equation
- Write the standard form equation of an ellipse given key features
- Find the foci (foci) of an ellipse using the relationship between a, b, and c

Problems

1. Identify the center and state whether the major axis is along the x-axis or y-axis for the ellipse with the equation below.

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

2. For the ellipse given below, find the values of a and b, then list all four vertices.

$$\frac{x^2}{49} + \frac{y^2}{16} = 1$$

3. An ellipse is centered at the origin with vertices at (0, 6), (0, –6), (4, 0), and (–4, 0). Write the standard form equation of this ellipse.

4. Rewrite the equation below in standard form by dividing both sides by 225, then identify a^2 and b^2.

$$25x^2 + 9y^2 = 225$$

5. Find the foci of the ellipse whose equation is given below. Use the relationship $c^2 = b^2 - a^2$ where b^2 is the larger denominator.

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

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6. Find the foci of the ellipse with the equation below.

$$\frac{x^2}{49} + \frac{y^2}{24} = 1$$

7. Write the standard form equation of an ellipse centered at (3, –2) with a = 5 along the x-direction and b = 3 along the y-direction.

8. Convert the equation below to standard form by completing the square, then identify the center, a, and b.

$$4x^2 + 9y^2 - 16x + 36y + 16 = 0$$

9. An ellipse is centered at (–1, 4) with foci at (–1, 4 + 2√5) and (–1, 4 – 2√5), and b = 3 (minor semi-axis along the x-direction). Find a and write the standard equation.

10. Find all key features — center, a, b, c, vertices along both axes, and both foci — for the ellipse with the equation below.

$$\frac{(x+2)^2}{36} + \frac{(y-5)^2}{100} = 1$$

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Analyzing Ellipses: Standard Form, Vertices & Foci — Answer Key

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Answer Key

1. Answer: Center: (0, 0); major axis along the y-axis

- The equation is already in standard form with $h = 0$ and $k = 0$, so the center is (0, 0).
- Compare denominators: $25 > 9$, and 25 is under y^2, so the major axis is along the y-axis.

2. Answer: $a = 7$, $b = 4$; vertices: (7, 0), (–7, 0), (0, 4), (0, –4)

- Since $49 > 16$ and 49 is under x^2, the major axis is along the x-axis, so $a^2 = 49 \rightarrow a = 7$.
- $b^2 = 16 \rightarrow b = 4$.
- Major-axis vertices: (±7, 0); minor-axis vertices: (0, ±4).

3. Answer: $x^2/16 + y^2/36 = 1$

- The vertices along the y-axis are (0, ±6), so $b = 6$ (major axis along y-axis in this orientation).
- The vertices along the x-axis are (±4, 0), so $a = 4$.
- Standard form: $x^2/16 + y^2/36 = 1$.

4. Answer: $x^2/9 + y^2/25 = 1$; $a^2 = 9$, $b^2 = 25$

- Divide every term by 225: $25x^2/225 + 9y^2/225 = 1$.
- Simplify: $x^2/9 + y^2/25 = 1$.
- Therefore $a^2 = 9$ (under x^2) and $b^2 = 25$ (under y^2).

5. Answer: Foci: (0, 4) and (0, –4)

- Since $25 > 9$, the major axis is along the y-axis. $b^2 = 25$, $a^2 = 9$.
- $c^2 = b^2 - a^2 = 25 - 9 = 16$, so $c = 4$.
- Foci lie along the y-axis: (0, 4) and (0, –4).

6. Answer: Foci: (5, 0) and (–5, 0)

- Since $49 > 24$, the major axis is along the x-axis. $a^2 = 49$, $b^2 = 24$.
- $c^2 = a^2 - b^2 = 49 - 24 = 25$, so $c = 5$.
- Foci lie along the x-axis: (5, 0) and (–5, 0).

7. Answer: $(x - 3)^2/25 + (y + 2)^2/9 = 1$

- Center is $(h, k) = (3, -2)$, $a = 5$ (along x-axis), $b = 3$ (along y-axis).
- Substitute into standard form: $(x - 3)^2/a^2 + (y - (-2))^2/b^2 = 1$.
- Result: $(x - 3)^2/25 + (y + 2)^2/9 = 1$.

8. Answer: $(x - 2)^2/9 + (y + 2)^2/4 = 1$; center (2, –2), $a = 3$, $b = 2$

- Group x and y terms: $(4x^2 - 16x) + (9y^2 + 36y) = -16$.
- Factor: $4(x^2 - 4x) + 9(y^2 + 4y) = -16$.

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- Complete the square: $4(x-2)^2 - 16 + 9(y+2)^2 - 36 = -16 \rightarrow 4(x-2)^2 + 9(y+2)^2 = 36$.
- Divide by 36: $(x-2)^2/9 + (y+2)^2/4 = 1$. Center $(2, -2)$, $a = 3$, $b = 2$.

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9. Answer: $a = 7$; $(x + 1)^2/9 + (y - 4)^2/49 = 1$

- The foci are along the y-direction, so the major axis is vertical. $c = 2\sqrt{5}$, $b = 3$.
- Use $c^2 = a^2 - b^2$: $(2\sqrt{5})^2 = a^2 - 9 \rightarrow 20 = a^2 - 9 \rightarrow a^2 = 29$. Wait — re-check: $c = 2\sqrt{5}$ means $c^2 = 20$, $b = 3$ means $b^2 = 9$, so $a^2 = 20 + 9 = 29$ and $a = \sqrt{29}$.
- Correction note: $a^2 = 29$, $a = \sqrt{29}$.
- Standard form: $(x + 1)^2/9 + (y - 4)^2/29 = 1$.

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10. Answer: Center $(-2, 5)$; $a = 6$ (x), $b = 10$ (y); $c = 8$; vertices: $(4,5),(-8,5),(-2,15),(-2,-5)$; foci: $(-2,13),(-2,-3)$

- Center: $h = -2$, $k = 5 \rightarrow (-2, 5)$.
- Since $100 > 36$, major axis is along y-direction. $b^2 = 100 \rightarrow b = 10$; $a^2 = 36 \rightarrow a = 6$.
- $c^2 = b^2 - a^2 = 100 - 36 = 64 \rightarrow c = 8$.
- Major-axis vertices (along y): $(-2, 5 \pm 10) = (-2, 15)$ and $(-2, -5)$.
- Minor-axis vertices (along x): $(-2 \pm 6, 5) = (4, 5)$ and $(-8, 5)$.
- Foci (along y): $(-2, 5 \pm 8) = (-2, 13)$ and $(-2, -3)$.

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