Standard Equation of an Ellipse Given Its Parts

Pre-Calculus / Analytic Geometry Worksheet · Grade 10–12

Name: ________________________________ Date: ____________________

Learning Objectives

- Identify the center, foci, and values of a and b from given parts of an ellipse
- Write the standard equation of an ellipse centered at the origin or at (h, k)
- Convert the general form of an ellipse to standard form by completing the square

Problems

1. An ellipse is centered at the origin with foci at (2, 0) and (-2, 0), and a major axis of length 10. Find the value of a.

$$2a = 10$$

2. An ellipse centered at the origin has foci on the x-axis at (3, 0) and (-3, 0). The major axis has length 8. Find b squared.

$$c^2 = a^2 - b^2$$

3. Write the standard equation of an ellipse centered at the origin whose major axis lies along the x-axis, with a = 5 and b squared = 7.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

4. An ellipse centered at the origin has foci on the y-axis at (0, 4) and (0, -4), and a major axis of length 10. Write its standard equation.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

5. An ellipse has foci at (0, 1) and (4, 1), and a major axis of length 6. Find the center and the value of c.

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6. Using the ellipse from Problem 5 with center (2, 1), c = 2, and major axis of length 6, find the standard equation of the ellipse.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

7. An ellipse has foci at (1, -3) and (1, 5), and a minor axis of length 6. Write the standard equation of the ellipse.

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

8. Convert the general equation to standard form by completing the square. The equation is x squared plus 4 y squared plus 6x minus 8y plus 9 equals 0.

$$x^2 + 4y^2 + 6x - 8y + 9 = 0$$

9. Convert the general equation to standard form by completing the square, then identify the center, a, and b. The equation is 9x squared plus 4y squared minus 36x plus 8y plus 4 equals 0.

$$9x^2 + 4y^2 - 36x + 8y + 4 = 0$$

10. An ellipse in standard position has vertices at (7, 2) and (-3, 2), and co-vertices at (2, 5) and (2, -1). Write the standard equation and identify the foci.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

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Standard Equation of an Ellipse Given Its Parts — Answer Key

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Answer Key

1. Answer: $a = 5$

- The major axis length equals 2a, so $2a = 10$.
- Divide both sides by 2: $a = 5$.

2. Answer: $b^2 = 7$

- From the major axis: $2a = 8$, so $a = 4$ and $a^2 = 16$.
- The distance from center to each focus is $c = 3$, so $c^2 = 9$.
- Using $c^2 = a^2 - b^2$: $9 = 16 - b^2$, so $b^2 = 7$.

3. Answer: $x^2/25 + y^2/7 = 1$

- Since the major axis is along the x-axis, a^2 goes under x^2.
- $a = 5$, so $a^2 = 25$; $b^2 = 7$.
- Substitute: $x^2/25 + y^2/7 = 1$.

4. Answer: $x^2/9 + y^2/25 = 1$

- Major axis is along the y-axis, so a^2 is under y^2.
- $2a = 10$, so $a = 5$ and $a^2 = 25$.
- $c = 4$, $c^2 = 16$; $b^2 = a^2 - c^2 = 25 - 16 = 9$.
- Standard equation: $x^2/9 + y^2/25 = 1$.

5. Answer: Center = (2, 1); c = 2

- The center is the midpoint of the two foci: $((0+4)/2, (1+1)/2) = (2, 1)$.
- The distance from the center to each focus is $c = |2 - 0| = 2$.

6. Answer: $(x-2)^2/9 + (y-1)^2/5 = 1$

- Center is $(h, k) = (2, 1)$.
- $2a = 6$, so $a = 3$ and $a^2 = 9$.
- $c = 2$, $c^2 = 4$; $b^2 = a^2 - c^2 = 9 - 4 = 5$.
- The foci share $y = 1$, so the major axis is horizontal; a^2 goes under $(x-2)^2$.
- Standard equation: $(x-2)^2/9 + (y-1)^2/5 = 1$.

7. Answer: $(x-1)^2/9 + (y-1)^2/25 = 1$

- Center is the midpoint of the foci: $((1+1)/2, (-3+5)/2) = (1, 1)$.
- c = distance from center to focus $= |5 - 1| = 4$.
- Minor axis length $= 2b = 6$, so $b = 3$ and $b^2 = 9$.
- $a^2 = b^2 + c^2 = 9 + 16 = 25$.
- Since the foci share $x = 1$, the major axis is vertical; a^2 goes under $(y-1)^2$.

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- Standard equation: $(x-1)^2/9 + (y-1)^2/25 = 1$.

8. Answer: $(x+3)^2/4 + (y-1)^2/1 = 1$

- Group x and y terms and move constant: $(x^2 + 6x) + 4(y^2 - 2y) = -9$.
- Complete the square for x: half of 6 is 3; $3^2 = 9$. Add 9 to both sides.
- Complete the square for y inside the parentheses: half of -2 is -1; $(-1)^2 = 1$. Add $4\times1 = 4$ to both sides.
- Result: $(x+3)^2 + 4(y-1)^2 = -9 + 9 + 4 = 4$.
- Divide every term by 4: $(x+3)^2/4 + (y-1)^2/1 = 1$.

9. Answer: $(x-2)^2/4 + (y+1)^2/9 = 1$; Center (2,–1), a = 3, b = 2

- Group and move constant: $9(x^2 - 4x) + 4(y^2 + 2y) = -4$.
- Complete the square for x: $(x-2)^2$, add $9\times4 = 36$ to right side.
- Complete the square for y: $(y+1)^2$, add $4\times1 = 4$ to right side.
- Result: $9(x-2)^2 + 4(y+1)^2 = -4 + 36 + 4 = 36$.
- Divide by 36: $(x-2)^2/4 + (y+1)^2/9 = 1$.
- Center: (2, -1); since 9 > 4, $a^2 = 9$ (under y term), $b^2 = 4$.

10. Answer: $(x-2)^2/25 + (y-2)^2/9 = 1$; Foci at (2±4, 2) = (6, 2) and (–2, 2)

- Center is the midpoint of the vertices: ((7+(-3))/2, (2+2)/2) = (2, 2).
- a = distance from center to vertex along major axis: |7 - 2| = 5, so $a^2 = 25$.
- b = distance from center to co-vertex: |5 - 2| = 3, so $b^2 = 9$.
- The major axis is horizontal (vertices share y = 2), so a^2 goes under $(x-2)^2$.
- Standard equation: $(x-2)^2/25 + (y-2)^2/9 = 1$.
- $c^2 = a^2 - b^2 = 25 - 9 = 16$, so c = 4.
- Foci are at (2 ± 4, 2) = (6, 2) and (–2, 2).

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