Graphing & Analyzing Hyperbolas

Conic Sections Worksheet · Algebra 2 / Pre-Calculus · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify the standard form of a hyperbola and distinguish it from an ellipse
- Extract key features (center, vertices, foci, asymptotes) from the standard equation
- Graph a hyperbola by constructing the rectangle framework and asymptotes

Problems

1. Identify whether the equation represents an ellipse or a hyperbola. State how you know.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

2. Find the values of a and b for the hyperbola, then state the length of the transverse axis.

$$\frac{x^2}{25} - \frac{y^2}{49} = 1$$

3. State the center, and identify whether the hyperbola opens horizontally or vertically.

$$\frac{(y-3)^2}{4} - \frac{(x+1)^2}{9} = 1$$

4. Find the vertices of the hyperbola whose center is at the origin.

$$\frac{x^2}{36} - \frac{y^2}{16} = 1$$

5. Use the relationship between a, b, and c to find the foci of the hyperbola centered at the origin.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

6. Write the equations of the asymptotes for the hyperbola centered at the origin.

Scan to watch

$$\frac{y^2}{25} - \frac{x^2}{4} = 1$$

7. Find the center, vertices, and foci of the hyperbola below.

$$\frac{(x-2)^2}{16} - \frac{(y+3)^2}{9} = 1$$

8. Describe all steps needed to graph the hyperbola by constructing the rectangle framework and asymptotes, then identify how the hyperbola opens.

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

9. Write the standard equation of the hyperbola with the given characteristics: center at (0, 0), a vertex at (0, 4), and a focus at (0, 5). Determine whether it opens horizontally or vertically.

10. Write the equation of the hyperbola in standard form by completing the square, then identify the center, vertices, foci, and equations of the asymptotes.

$$9x^2 - 4y^2 - 18x + 16y - 43 = 0$$

Graphing & Analyzing Hyperbolas — Answer Key

Conic Sections Worksheet · Algebra 2 / Pre-Calculus · Grade 10–12

Answer Key

1. Answer: Hyperbola — the terms are separated by a minus sign

- An ellipse has the form $x^2/a^2 + y^2/b^2 = 1$ (plus sign between terms).
- A hyperbola has the form $x^2/a^2 - y^2/b^2 = 1$ (minus sign between terms).
- Because the sign between the two fractions is minus, this is a hyperbola.

2. Answer: a = 5, b = 7; transverse axis length = 10

- The denominator under x^2 is a^2, so $a^2 = 25$, giving $a = 5$.
- The denominator under y^2 is b^2, so $b^2 = 49$, giving $b = 7$.
- The transverse axis runs between the two vertices along the x-axis: length = $2a = 10$.

3. Answer: Center (-1, 3); opens vertically

- The standard form with y^2 as the positive term indicates vertical opening.
- The center is (h, k) where the equation is written as $(y - k)^2/a^2 - (x - h)^2/b^2 = 1$.
- Here $h = -1$ and $k = 3$, so center = (–1, 3).

4. Answer: Vertices at (6, 0) and (–6, 0)

- The positive term is $x^2/36$, so the hyperbola opens horizontally.
- $a^2 = 36$, so $a = 6$.
- Vertices are located a units from the center along the x-axis: (±6, 0).

5. Answer: Foci at (5, 0) and (–5, 0)

- For a hyperbola, $c^2 = a^2 + b^2$.
- Here $a^2 = 9$ and $b^2 = 16$, so $c^2 = 9 + 16 = 25$.
- $c = \sqrt{25} = 5$.
- Since the hyperbola opens horizontally, foci are at (±5, 0).

6. Answer: y = (5/2)x and y = –(5/2)x

- For a vertically opening hyperbola $y^2/a^2 - x^2/b^2 = 1$, the asymptotes are $y = \pm(a/b)x$.
- Here $a^2 = 25$ so $a = 5$, and $b^2 = 4$ so $b = 2$.
- Asymptotes: $y = \pm(5/2)x$.

7. Answer: Center (2, –3); Vertices (6, –3) and (–2, –3); Foci (7, –3) and (–3, –3)

- Center is (h, k) = (2, –3).
- $a^2 = 16$, $a = 4$; vertices are h ± a units along x-axis: (2+4, –3)=(6,–3) and (2–4,–3)=(–2,–3).
- $b^2 = 9$, $b = 3$; $c^2 = a^2 + b^2 = 16 + 9 = 25$, $c = 5$.
- Foci are h ± c units along x-axis: (2+5,–3)=(7,–3) and (2–5,–3)=(–3,–3).

8. Answer: Opens horizontally; rectangle corners at (±2, ±3); asymptotes y = (3/2)x and y = –(3/2)x

- Step 1 — Framework: $a^2 = 4$ so $a = 2$ (units along x-axis); $b^2 = 9$ so $b = 3$ (units along y-axis).

Scan to watch

- Plot points (2, 0), (–2, 0) on x-axis and (0, 3), (0, –3) on y-axis.
- Draw a rectangle through these four points.
- Step 2 — Asymptotes: Draw the diagonals of the rectangle; these are the asymptotes $y = \pm(3/2)x$.
- Step 3 — Graph: Because the positive term is x^2, the hyperbola opens left and right, with vertices at (±2, 0), curving toward the asymptotes.

---

9. Answer: $y^2/16 - x^2/9 = 1$; opens vertically

- Vertex at (0, 4) means a = 4 and the transverse axis is vertical, so the y^2 term is positive.
- Focus at (0, 5) means c = 5.
- Using $c^2 = a^2 + b^2$: $25 = 16 + b^2$, so $b^2 = 9$.
- Standard form: $y^2/16 - x^2/9 = 1$.

---

10. Answer: Center (1, 2); Vertices $(1\pm2\sqrt{(1)}, 2)$ — i.e. (3,2) & (–1,2); Foci $(1\pm\sqrt{13}, 2)$; Asymptotes $y-2 = \pm(3/2)(x-1)$

- Group x and y terms: $(9x^2 - 18x) - (4y^2 - 16y) = 43$.
- Factor: $9(x^2 - 2x) - 4(y^2 - 4y) = 43$.
- Complete the square: $9(x-1)^2 - 9 - 4(y-2)^2 + 16 = 43$.
- Simplify: $9(x-1)^2 - 4(y-2)^2 = 43 + 9 - 16 = 36$.
- Divide by 36: $(x-1)^2/4 - (y-2)^2/9 = 1$.
- Center (1, 2); $a^2 = 4$, a = 2; $b^2 = 9$, b = 3; $c^2 = 4+9 = 13$, $c = \sqrt{13}$.
- Vertices: (1±2, 2) = (3, 2) and (–1, 2); Foci: $(1\pm\sqrt{13}, 2)$.
- Asymptotes: $y - 2 = \pm(b/a)(x - 1) = \pm(3/2)(x - 1)$.

Scan to watch