Graphing and Analyzing Hyperbolas

Conic Sections Worksheet · Grade 10–12

Name: ______________________________ Date: __________________

Learning Objectives

- Identify the standard form of a hyperbola and distinguish it from an ellipse
- Extract key features of a hyperbola including center, vertices, foci, and asymptotes
- Graph a hyperbola by constructing the rectangular framework and asymptotes

Problems

1. Identify whether the following equation represents a hyperbola or an ellipse, and explain why.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

2. Identify the center, the value of a, and the value of b for the hyperbola given below.

$$\frac{x^2}{25} - \frac{y^2}{49} = 1$$

3. Find the vertices of the hyperbola whose equation is given below. The hyperbola opens horizontally.

$$\frac{x^2}{36} - \frac{y^2}{4} = 1$$

4. Find the foci of the hyperbola whose equation is given below using the relationship c squared equals a squared plus b squared.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

5. Write the equations of the asymptotes for the hyperbola given below. Recall that asymptotes pass through the corners of the rectangular framework.

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

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6. Determine the center, vertices, and direction of opening for the hyperbola given below, where h and k are the shifts from the origin.

$$\frac{(x-3)^2}{16} - \frac{(y+2)^2}{9} = 1$$

7. Find the foci of the hyperbola with the equation given below. The hyperbola is centered at (2, –1).

$$\frac{(y+1)^2}{25} - \frac{(x-2)^2}{144} = 1$$

8. Rewrite the equation below in standard form by completing the square, then identify the center and values of a and b.

$$9x^2 - 4y^2 - 18x + 16y - 43 = 0$$

9. Write the standard equation of a hyperbola centered at the origin that opens vertically, with vertices at (0, 5) and (0, –5) and foci at (0, 13) and (0, –13).

10. A hyperbola has foci at (0, 10) and (0, –10) and the difference of distances from any point on the hyperbola to the two foci is 12. Write the standard equation of the hyperbola and find the equations of its asymptotes.

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Graphing and Analyzing Hyperbolas — Answer Key

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Answer Key

1. Answer: Hyperbola, because the terms are subtracted (minus sign between terms)

- An ellipse has the form $(x^2/a^2) + (y^2/b^2) = 1$ with a plus sign.
- A hyperbola has the form $(x^2/a^2) – (y^2/b^2) = 1$ with a minus sign.
- Since the equation has a minus sign between the two fractions, it is a hyperbola.

2. Answer: Center: (0, 0), a = 5, b = 7

- The equation is in standard form with $h = 0$ and $k = 0$, so the center is (0, 0).
- The denominator under x^2 is 25, so $a^2 = 25$ and $a = 5$.
- The denominator under y^2 is 49, so $b^2 = 49$ and $b = 7$.

3. Answer: Vertices: (6, 0) and (–6, 0)

- The hyperbola opens horizontally because the x^2 term is positive.
- $a^2 = 36$, so $a = 6$.
- For a horizontally opening hyperbola centered at the origin, vertices are at $(\pm a, 0)$ = (6, 0) and (–6, 0).

4. Answer: Foci: (5, 0) and (–5, 0)

- $a^2 = 9$ and $b^2 = 16$.
- Use $c^2 = a^2 + b^2 = 9 + 16 = 25$.
- So $c = \sqrt{25} = 5$.
- Since the hyperbola opens horizontally, foci are at (±5, 0).

5. Answer: y = (3/2)x and y = –(3/2)x

- For a hyperbola of the form $x^2/a^2 – y^2/b^2 = 1$, asymptotes are $y = \pm(b/a)x$.
- Here $a^2 = 4$ so $a = 2$, and $b^2 = 9$ so $b = 3$.
- Asymptotes: $y = (3/2)x$ and $y = –(3/2)x$.

6. Answer: Center: (3, –2); Vertices: (7, –2) and (–1, –2); Opens horizontally

- The center is at (h, k) = (3, –2).
- $a^2 = 16$, so $a = 4$.
- Since x^2 term is positive, the hyperbola opens horizontally.
- Vertices are at $(h \pm a, k) = (3 \pm 4, –2)$, giving (7, –2) and (–1, –2).

7. Answer: Foci: (2, 12) and (2, –14)

- The y^2 term is positive, so the hyperbola opens vertically.
- $a^2 = 25$ and $b^2 = 144$.
- $c^2 = a^2 + b^2 = 25 + 144 = 169$, so $c = 13$.
- Center is (2, –1). Foci are at $(h, k \pm c) = (2, –1 \pm 13)$, giving (2, 12) and (2, –14).

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8. Answer: $(x-1)^2/4 - (y-2)^2/9 = 1$; Center: (1, 2), a = 2, b = 3

- Group x and y terms: $9(x^2 - 2x) - 4(y^2 - 4y) = 43$.
- Complete the square: $9(x^2 - 2x + 1) - 4(y^2 - 4y + 4) = 43 + 9 - 16$.
- Simplify: $9(x-1)^2 - 4(y-2)^2 = 36$.
- Divide by 36: $(x-1)^2/4 - (y-2)^2/9 = 1$.
- Center: (1, 2), $a^2 = 4$ so a = 2, $b^2 = 9$ so b = 3.

9. Answer: $y^2/25 - x^2/144 = 1$

- Since the hyperbola opens vertically, the y^2 term is positive: $y^2/a^2 - x^2/b^2 = 1$.
- Vertices are at (0, ±5), so a = 5 and $a^2 = 25$.
- Foci are at (0, ±13), so c = 13.
- Use $c^2 = a^2 + b^2$: $169 = 25 + b^2$, so $b^2 = 144$.
- Standard equation: $y^2/25 - x^2/144 = 1$.

10. Answer: $y^2/36 - x^2/64 = 1$; Asymptotes: $y = (3/4)x$ and $y = -(3/4)x$

- The foci are on the y-axis so the hyperbola opens vertically: $y^2/a^2 - x^2/b^2 = 1$.
- c = 10 since foci are at (0, ±10).
- The constant difference of distances equals 2a = 12, so a = 6 and $a^2 = 36$.
- Use $c^2 = a^2 + b^2$: $100 = 36 + b^2$, so $b^2 = 64$ and b = 8.
- Standard equation: $y^2/36 - x^2/64 = 1$.
- Asymptotes for a vertical hyperbola: $y = \pm(a/b)x = \pm(6/8)x = \pm(3/4)x$.