Converting Conic Sections to Standard Form

Completing the Square · Algebra Worksheet · Grade 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Identify the type of conic section from its general equation
- Apply the completing the square technique to rewrite conic equations in standard form
- Extract key features such as vertex and center from the standard form of a conic section

Problems

1. Complete the square for the expression below and write it as a perfect square binomial squared.

$$y^2 + 8y$$

2. Complete the square for the expression below and write it as a perfect square binomial squared.

$$x^2 - 10x$$

3. Identify the conic section represented by the equation below, then rewrite it in standard form by completing the square.

$$y^2 - 4y - 8x + 20 = 0$$

4. Identify the conic section and convert the equation below to standard form by completing the square.

$$y^2 + 6y - 4x + 1 = 0$$

5. Identify the conic section and convert the equation below to standard form by completing the square.

$$x^2 + 4x - 12y - 8 = 0$$

6. Identify the conic section and convert the equation below to standard form by completing the square. Then state the center.

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$$x^2 + y^2 + 6x - 4y - 3 = 0$$

7. Identify the conic section and convert the equation below to standard form by completing the square. Then state the center.

$$x^2 + 4y^2 + 6x - 8y - 3 = 0$$

8. Identify the conic section and convert the equation below to standard form by completing the square. Then state the center.

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$

9. Identify the conic section and convert the equation below to standard form by completing the square. Then state the center.

$$4x^2 - 9y^2 + 16x + 18y - 29 = 0$$

10. Identify the conic section and convert the equation below to standard form by completing the square. State the center and identify the type of conic.

$$16x^2 - 4y^2 - 64x + 24y - 36 = 0$$

Converting Conic Sections to Standard Form — Answer Key

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Answer Key

1. Answer: $(y + 4)^2 - 16$

- Take half of the coefficient of y: $8 \div 2 = 4$
- Square it: $4^2 = 16$
- Add and subtract 16: $y^2 + 8y + 16 - 16$
- Factor the trinomial: $(y + 4)^2 - 16$

2. Answer: $(x - 5)^2 - 25$

- Take half of –10: $-10 \div 2 = -5$
- Square it: $(-5)^2 = 25$
- Add and subtract 25: $x^2 - 10x + 25 - 25$
- Factor: $(x - 5)^2 - 25$

3. Answer: $(y - 2)^2 = 8(x - 2)$, parabola opening right, vertex (2, 2)

- Only y is squared, so this is a parabola
- Group y terms and move others to the right: $y^2 - 4y = 8x - 20$
- Complete the square: half of –4 is –2, $(-2)^2 = 4$; add 4 to both sides
- Left side: $(y - 2)^2$; right side: $8x - 20 + 4 = 8x - 16$
- Factor right side: $8(x - 2)$
- Standard form: $(y - 2)^2 = 8(x - 2)$; vertex is (2, 2)

4. Answer: $(y + 3)^2 = 4(x + 2)$, parabola opening right, vertex (–2, –3)

- Only y is squared, so this is a parabola
- Rearrange: $y^2 + 6y = 4x - 1$
- Half of 6 is 3; $3^2 = 9$; add 9 to both sides
- Left side: $(y + 3)^2$; right side: $4x - 1 + 9 = 4x + 8$
- Factor: $4(x + 2)$
- Standard form: $(y + 3)^2 = 4(x + 2)$; vertex (–2, –3)

5. Answer: $(x + 2)^2 = 12(y + 1)$, parabola opening upward, vertex (–2, –1)

- Only x is squared, so this is a parabola (opens up or down)
- Rearrange: $x^2 + 4x = 12y + 8$
- Half of 4 is 2; $2^2 = 4$; add 4 to both sides
- Left side: $(x + 2)^2$; right side: $12y + 8 + 4 = 12y + 12$
- Factor: $12(y + 1)$
- Standard form: $(x + 2)^2 = 12(y + 1)$; vertex (–2, –1)

6. Answer: $(x + 3)^2 + (y - 2)^2 = 16$, circle with center (–3, 2) and radius 4

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- Both variables are squared with equal coefficients, so this is a circle
- Group: $(x^2 + 6x) + (y^2 - 4y) = 3$
- For x: half of 6 = 3, $3^2 = 9$; for y: half of –4 = –2, $(-2)^2 = 4$
- Add 9 and 4 to both sides: $(x^2 + 6x + 9) + (y^2 - 4y + 4) = 3 + 9 + 4 = 16$
- Factor: $(x + 3)^2 + (y - 2)^2 = 16$
- Center (–3, 2), radius = 4

7. Answer: $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{4} = 1$, ellipse with center (–3, 1)

- Both variables squared with different positive coefficients → ellipse
- Group: $(x^2 + 6x) + 4(y^2 - 2y) = 3$
- For x: half of 6 = 3, $3^2 = 9$; add 9 to both sides
- For y group: half of –2 = –1, $(-1)^2 = 1$; add 4·1 = 4 to both sides
- $(x + 3)^2 + 4(y - 1)^2 = 3 + 9 + 4 = 16$
- Divide all terms by 16: $(x+3)^2/16 + (y-1)^2/4 = 1$; center (–3, 1)

8. Answer: $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$, ellipse with center (1, –2)

- Both variables squared with different positive coefficients → ellipse
- Group: $9(x^2 - 2x) + 4(y^2 + 4y) = 11$
- For x group: half of –2 = –1, $(-1)^2 = 1$; add 9·1 = 9 to both sides
- For y group: half of 4 = 2, $2^2 = 4$; add 4·4 = 16 to both sides
- $9(x - 1)^2 + 4(y + 2)^2 = 11 + 9 + 16 = 36$
- Divide by 36: $(x-1)^2/4 + (y+2)^2/9 = 1$; center (1, –2)

9. Answer: $\frac{(x+2)^2}{9} - \frac{(y-1)^2}{4} = 1$, hyperbola with center (–2, 1)

- Variables squared with opposite signs → hyperbola
- Group: $4(x^2 + 4x) - 9(y^2 - 2y) = 29$
- For x group: half of 4 = 2, $2^2 = 4$; add 4·4 = 16 to both sides
- For y group: half of –2 = –1, $(-1)^2 = 1$; subtract 9·1 = 9 from both sides (note the negative sign)
- $4(x + 2)^2 - 9(y - 1)^2 = 29 + 16 - 9 = 36$
- Divide by 36: $(x+2)^2/9 - (y-1)^2/4 = 1$; hyperbola center (–2, 1)

10. Answer: $\frac{(x-2)^2}{4} - \frac{(y-3)^2}{16} = 1$, hyperbola with center (2, 3)

- Variables squared with opposite signs → hyperbola
- Group: $16(x^2 - 4x) - 4(y^2 - 6y) = 36$
- For x group: half of –4 = –2, $(-2)^2 = 4$; add 16·4 = 64 to both sides
- For y group: half of –6 = –3, $(-3)^2 = 9$; subtract 4·9 = 36 from right (adding to left inside negative group adds –36 outside)
- $16(x - 2)^2 - 4(y - 3)^2 = 36 + 64 - 36 = 64$
- Divide by 64: $(x-2)^2/4 - (y-3)^2/16 = 1$; hyperbola center (2, 3)

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