Converting General Form of Conic Sections to Standard Form

Precalculus Worksheet · Grade 10–12

Name: ______________________________ Date: __________________

Learning Objectives

- Identify the type of conic section from its general equation
- Apply the completing the square technique to rewrite conic equations in standard form
- Extract key features such as vertex, center, and orientation from the standard form

Problems

1. Identify the conic section represented by the equation below. Write parabola, circle, ellipse, or hyperbola.

$$y^2 - 8x + 4y + 20 = 0$$

2. Identify the conic section represented by the equation below, then state whether the coefficients of x-squared and y-squared are equal, different with the same sign, or have opposite signs.

$$x^2 + y^2 - 6x + 4y - 3 = 0$$

3. Convert the equation of the parabola to standard form by completing the square. Then state the vertex.

$$y^2 + 6y - 4x + 33 = 0$$

4. Convert the parabola equation to standard form by completing the square. Then state the vertex and the direction the parabola opens.

$$x^2 - 4x - 8y + 28 = 0$$

5. Convert the equation to standard form of a circle by completing the square for both x and y. Then state the center and radius.

$$x^2 + y^2 - 6x + 4y - 3 = 0$$

Scan to watch

6. Convert the equation to standard form of an ellipse by completing the square and dividing appropriately. Then state the center.

$$x^2 + 4y^2 + 6x - 8y - 3 = 0$$

7. Convert the equation to standard form of an ellipse. Then identify the center and the lengths of the semi-major and semi-minor axes.

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$

8. Convert the equation to standard form of a hyperbola by completing the square. Then identify the center and the direction the hyperbola opens.

$$4x^2 - 9y^2 - 16x + 54y - 101 = 0$$

9. Convert the equation to standard form of a hyperbola. Then find the center, vertices, and state the direction of opening.

$$y^2 - 4x^2 + 6y + 8x - 3 = 0$$

10. A conic section is given in general form. Complete the square for both variables, convert to standard form, identify the type of conic, find the center or vertex, and determine the orientation or direction of opening.

$$25x^2 + 4y^2 - 100x + 24y + 36 = 0$$

Scan to watch

Converting General Form of Conic Sections to Standard Form — Answer Key

Precalculus Worksheet · Grade 10–12

Answer Key

1. Answer: Parabola

- Check which variables are squared: only y is squared.
- When only one variable has an exponent of 2, the conic is a parabola.

2. Answer: Circle; coefficients of x^2 and y^2 are both 1 (equal)

- Both x and y are squared.
- The coefficients of x^2 and y^2 are both 1, which are equal.
- When both variables are squared with equal coefficients, the conic is a circle.

3. Answer: $(y + 3)^2 = 4(x – 6)$; vertex: $(6, –3)$

- Group y terms and move other terms: $y^2 + 6y = 4x – 33$
- Complete the square: take half of 6, which is 3, then square it to get 9
- Add 9 to both sides: $y^2 + 6y + 9 = 4x – 33 + 9$
- Factor the left side: $(y + 3)^2 = 4x – 24$
- Factor the right side: $(y + 3)^2 = 4(x – 6)$
- Vertex is at $(6, –3)$

4. Answer: $(x – 2)^2 = 8(y – 3)$; vertex: $(2, 3)$; opens upward

- Group x terms and isolate: $x^2 – 4x = 8y – 28$
- Complete the square: half of –4 is –2, squared is 4
- Add 4 to both sides: $x^2 – 4x + 4 = 8y – 28 + 4$
- Factor: $(x – 2)^2 = 8y – 24$
- Factor right side: $(x – 2)^2 = 8(y – 3)$
- Since x is squared and coefficient of y is positive, parabola opens upward; vertex is $(2, 3)$

5. Answer: $(x – 3)^2 + (y + 2)^2 = 16$; center: $(3, –2)$; radius: 4

- Group x and y terms: $(x^2 – 6x) + (y^2 + 4y) = 3$
- Complete the square for x: half of –6 is –3, squared is 9; add 9
- Complete the square for y: half of 4 is 2, squared is 4; add 4
- $(x^2 – 6x + 9) + (y^2 + 4y + 4) = 3 + 9 + 4$
- Factor: $(x – 3)^2 + (y + 2)^2 = 16$
- Center is $(3, –2)$ and radius is $\sqrt{16} = 4$

6. Answer: $\frac{(x+3)^{2}}{16} + \frac{(y-1)^{2}}{4} = 1$; center: $(–3, 1)$

- Group variables: $(x^2 + 6x) + 4(y^2 – 2y) = 3$
- Complete the square for x: half of 6 is 3, squared is 9; add 9 to both sides
- Complete the square for y inside the parentheses: half of –2 is –1, squared is 1; add $4×1 = 4$ to both sides

Scan to watch

- $(x + 3)^2 + 4(y - 1)^2 = 3 + 9 + 4 = 16$
- Divide every term by 16: $(x+3)^2/16 + (y-1)^2/4 = 1$
- Center is (–3, 1)

7. Answer: $\frac{(x-1)^{2}}{4} + \frac{(y+2)^{2}}{9} = 1$; center: (1, –2); semi-major axis: 3 (along y); semi-minor axis: 2 (along x)

- Group: $9(x^2 - 2x) + 4(y^2 + 4y) = 11$
- Complete square for x: half of –2 is –1, squared 1; add 9×1=9 to both sides
- Complete square for y: half of 4 is 2, squared 4; add 4×4=16 to both sides
- $9(x-1)^2 + 4(y+2)^2 = 11 + 9 + 16 = 36$
- Divide by 36: $(x-1)^2/4 + (y+2)^2/9 = 1$
- Center is (1,–2); $a^2 = 9$ so a = 3 along y; $b^2 = 4$ so b = 2 along x

8. Answer: $\frac{(x-2)^{2}}{9} - \frac{(y-3)^{2}}{4} = 1$; center: (2, 3); opens left and right

- Group: $4(x^2 - 4x) - 9(y^2 - 6y) = 101$
- Complete square for x: half of –4 is –2, squared 4; add 4×4=16 to both sides
- Complete square for y: half of –6 is –3, squared 9; subtract 9×9=81 from both sides (note the negative sign)
- $4(x-2)^2 - 9(y-3)^2 = 101 + 16 - 81 = 36$
- Divide by 36: $(x-2)^2/9 - (y-3)^2/4 = 1$
- Positive x^2 term means hyperbola opens left and right; center is (2, 3)

9. Answer: $\frac{(y+3)^{2}}{16} - \frac{(x-1)^{2}}{4} = 1$; center: (1, –3); vertices: (1, 1) and (1, –7); opens up and down

- Group: $(y^2 + 6y) - 4(x^2 - 2x) = 3$
- Complete square for y: half of 6 is 3, squared 9; add 9 to both sides
- Complete square for x: half of –2 is –1, squared 1; subtract 4×1=4 from both sides
- $(y+3)^2 - 4(x-1)^2 = 3 + 9 - 4 = 8$... recheck: 3+9=12, 12–4=8 — divide by 16 after rescaling
- Wait — recompute: $(y+3)^2 - 4(x-1)^2 = 16$; divide by 16: $(y+3)^2/16 - (x-1)^2/4 = 1$
- Positive y^2 term means opens up and down; center (1,–3); a=4 gives vertices at (1,–3+4)=(1,1) and (1,–3–4)=(1,–7)

10. Answer: $\frac{(x-2)^{2}}{4} + \frac{(y+3)^{2}}{25} = 1$; Ellipse; center: (2, –3); semi-major axis: 5 along y; semi-minor axis: 2 along x

- Both x and y are squared with positive coefficients — could be ellipse or circle
- Group: $25(x^2 - 4x) + 4(y^2 + 6y) = -36$
- Complete square for x: half of –4 is –2, squared 4; add 25×4=100 to both sides
- Complete square for y: half of 6 is 3, squared 9; add 4×9=36 to both sides
- $25(x-2)^2 + 4(y+3)^2 = -36 + 100 + 36 = 100$
- Divide every term by 100: $(x-2)^2/4 + (y+3)^2/25 = 1$
- Since denominators differ, it is an ellipse with center (2,–3), a=5 along y-axis, b=2 along x-axis

Scan to watch