Applications of Derivatives: Rectilinear Motion

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Identify position, velocity, and acceleration functions using derivative notation
- Compute total distance traveled and displacement of a particle over a time interval
- Determine when a particle changes direction by analyzing the velocity function

For each problem, use derivatives to analyze the motion of a particle and show all work in the space provided.

1. Given the position function below, find the velocity function v(t).

$$s(t) = t^3 - 6t^2 + 9t$$

Answer: _____

2. Find the times when the particle is at rest for s(t) = t^3 - 6t^2 + 9t.

$$v(t) = 3t^2 - 12t + 9 = 0$$

Answer: _____

3. Find the acceleration function a(t) for the position function s(t) = t^3 - 6t^2 + 9t.

$$a(t) = \frac{d^2s}{dt^2}$$

Answer: _____

4. Find the displacement of the particle on the interval from t = 0 to t = 5 seconds.

$$\text{Displacement} = s(5) - s(0)$$

Answer: _____

5. Find the total distance traveled by the particle during the first 5 seconds for s(t) = t^3 - 6t^2 + 9t.

$$D = |s(1) - s(0)| + |s(3) - s(1)| + |s(5) - s(3)|$$

Answer: _____

6. Find the speed of the particle at t = 4 seconds.

$$\text{Speed} = |v(4)|$$

Answer: _____

7. Determine whether the particle is moving forward or backward at t = 2 seconds.

$$v(2) = 3(2)^2 - 12(2) + 9$$

Answer: _____

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8. Find the acceleration at t = 3 seconds for s(t) = t^3 - 6t^2 + 9t.

$a(3) = 6(3) - 12$

Answer: _____

9. For s(t) = t^3 - 6t^2 + 9t, on which interval(s) is the particle moving backward during $0 \leq t \leq 5$?

$v(t) < 0$

Answer: _____

10. A particle has position s(t) = t^3 - 6t^2 + 9t. Find the total distance traveled during the first 3 seconds.

$D = |s(1) - s(0)| + |s(3) - s(1)|$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students that total distance requires breaking the interval at points where velocity equals zero, while displacement is simply s(b) - s(a).

Solutions

1. Given the position function below, find the velocity function v(t).

$$s(t) = t^3 - 6t^2 + 9t$$

→ *Take the derivative of s(t) with respect to t.*
→ *Apply the power rule to each term.*
→ *The result is the velocity function v(t) = 3t squared minus 12t plus 9.*

Answer: $v(t) = 3t^2 - 12t + 9$

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2. Find the times when the particle is at rest for s(t) = t^3 - 6t^2 + 9t.

$$v(t) = 3t^2 - 12t + 9 = 0$$

→ *Set the velocity function equal to zero.*
→ *Factor the quadratic as 3 times (t minus 1) times (t minus 3).*
→ *Apply the zero product property to find t equals 1 and t equals 3.*

Answer: $t = 1,\ t = 3$

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3. Find the acceleration function a(t) for the position function s(t) = t^3 - 6t^2 + 9t.

$$a(t) = \frac{d^2s}{dt^2}$$

→ *Differentiate the velocity function v(t) = 3t squared minus 12t plus 9.*
→ *Apply the power rule to each term.*
→ *The acceleration function is a(t) = 6t minus 12.*

Answer: $a(t) = 6t - 12$

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4. Find the displacement of the particle on the interval from t = 0 to t = 5 seconds.

$$\text{Displacement} = s(5) - s(0)$$

→ *Evaluate s(5) = 125 minus 150 plus 45 to get 20.*
→ *Evaluate s(0) = 0.*
→ *Subtract to get displacement of 20 meters.*

Answer: 20 meters

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5. Find the total distance traveled by the particle during the first 5 seconds for s(t) = t^3 - 6t^2 + 9t.

$$D = |s(1) - s(0)| + |s(3) - s(1)| + |s(5) - s(3)|$$

→ *Identify the critical points at t equals 1 and t equals 3 where velocity equals zero.*
→ *Compute s(0) = 0, s(1) = 4, s(3) = 0, s(5) = 20.*
→ *Sum the absolute differences: 4 plus 4 plus 20 equals 28 meters.*

Answer: 28 meters

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6. Find the speed of the particle at t = 4 seconds.

$\text{Speed} = |v(4)|$

→ *Substitute t equals 4 into v(t) = 3t squared minus 12t plus 9.*
→ *Compute 48 minus 48 plus 9 to get 9.*
→ *Take the absolute value to find the speed equals 9 meters per second.*

Answer: 9 m/s

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7. Determine whether the particle is moving forward or backward at t = 2 seconds.

$v(2) = 3(2)^2 - 12(2) + 9$

→ *Substitute t equals 2 into the velocity function.*
→ *Compute 12 minus 24 plus 9 to get negative 3.*
→ *Since velocity is negative, the particle is moving backward.*

Answer: $v(2) = -3,$ moving backward

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8. Find the acceleration at t = 3 seconds for s(t) = t^3 - 6t^2 + 9t.

$a(3) = 6(3) - 12$

→ *Substitute t equals 3 into the acceleration function a(t) = 6t minus 12.*
→ *Compute 18 minus 12.*
→ *The acceleration at t equals 3 is 6 meters per second squared.*

Answer: 6 m/s^2

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9. For s(t) = t^3 - 6t^2 + 9t, on which interval(s) is the particle moving backward during $0 \le t \le 5$?

$v(t) < 0$

→ *Use the factored velocity v(t) = 3(t minus 1)(t minus 3).*
→ *Test a value such as t equals 2 to see that v is negative.*
→ *Conclude the particle moves backward on the open interval from 1 to 3.*

Answer: $1 < t < 3$

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10. A particle has position s(t) = t^3 - 6t^2 + 9t. Find the total distance traveled during the first 3 seconds.

$D = |s(1) - s(0)| + |s(3) - s(1)|$

→ *Identify the critical point at t equals 1 within the interval from 0 to 3.*
→ *Compute s(0) = 0, s(1) = 4, s(3) = 0.*
→ *Sum absolute differences: 4 plus 4 equals 8 meters.*

Answer: 8 meters