



# Chain Rule: Derivatives of Composite Functions

Calculus Worksheet · Grade 11-12

Name: \_\_\_\_\_

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## Learning Objectives

- Identify composite functions of the form  $f(g(x))$
- Apply the chain rule to differentiate functions raised to a power
- Simplify derivatives obtained using the chain rule

Find the derivative of each composite function using the chain rule and simplify your answer.

### 1. Find the derivative using the chain rule.

$$a(x) = (5x + 1)^3$$

Answer: \_\_\_\_\_

### 2. Find the derivative using the chain rule.

$$m(x) = (3x - 2)^7$$

Answer: \_\_\_\_\_

### 3. Find the derivative using the chain rule.

$$b(x) = \sqrt{x + 1}$$

Answer: \_\_\_\_\_

### 4. Find the derivative using the chain rule.

$$c(x) = (4x)^{\frac{2}{3}}$$

Answer: \_\_\_\_\_

### 5. Find the derivative using the chain rule.

$$h(x) = (2x^2 + 3)^5$$

Answer: \_\_\_\_\_

### 6. Find the derivative using the chain rule.

$$p(x) = \sqrt{3x^2 - 5}$$

Answer: \_\_\_\_\_

### 7. Find the derivative using the chain rule.

$$q(x) = (x^3 + 2x)^4$$

Answer: \_\_\_\_\_



**8. Find the derivative using the chain rule.**

$$r(x) = (7x + 4)^{10}$$

Answer: \_\_\_\_\_

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**9. Find the derivative using the chain rule.**

$$s(x) = \sqrt[3]{6x - 1}$$

Answer: \_\_\_\_\_

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**10. Find the derivative using the chain rule.**

$$t(x) = (x^2 - 4x + 1)^6$$

Answer: \_\_\_\_\_





Encourage students to clearly identify the outer function  $f(x)$  and inner function  $g(x)$  before differentiating.

## Solutions

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1. Find the derivative using the chain rule.

$$a(x) = (5x + 1)^3$$

- Identify the outer function as the parentheses cubed and the inner function as  $5x$  plus  $1$ .
- Differentiate the outer function to get 3 times the parentheses squared.
- Multiply by the derivative of the inner function, which is  $5$ .
- Combine the constants  $3$  and  $5$  to get 15 times the quantity  $5x$  plus  $1$  squared.

**Answer:**  $a'(x) = 15(5x + 1)^2$

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2. Find the derivative using the chain rule.

$$m(x) = (3x - 2)^7$$

- Identify the outer function as the parentheses to the seventh power and the inner function as  $3x$  minus  $2$ .
- Differentiate the outer function to get 7 times the parentheses to the sixth power.
- Multiply by the derivative of the inner function, which is  $3$ .
- Combine the constants  $7$  and  $3$  to get 21 times the quantity  $3x$  minus  $2$  to the sixth power.

**Answer:**  $m'(x) = 21(3x - 2)^6$

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3. Find the derivative using the chain rule.

$$b(x) = \sqrt{x + 1}$$

- Rewrite the square root as the inside function raised to the one-half power.
- Differentiate the outer function using the power rule to get one-half times the parentheses to the negative one-half power.
- Multiply by the derivative of the inner function  $x$  plus  $1$ , which is  $1$ .
- Rewrite the negative exponent as a fraction with the square root in the denominator.

**Answer:**  $b'(x) = \frac{1}{2\sqrt{x + 1}}$

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4. Find the derivative using the chain rule.

$$c(x) = (4x)^{\frac{2}{3}}$$

- Identify the outer function as the parentheses raised to the two-thirds power.
- Differentiate the outer function to get two-thirds times the parentheses raised to the negative one-third power.
- Multiply by the derivative of the inner function  $4x$ , which is  $4$ .
- Combine the constants two-thirds and  $4$  to get eight-thirds times the quantity  $4x$  to the negative one-third.

**Answer:**  $c'(x) = \frac{8}{3}(4x)^{-\frac{1}{3}}$

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5. Find the derivative using the chain rule.

$$h(x) = (2x^2 + 3)^5$$

- Identify the outer function as the parentheses to the fifth power and the inner function as  $2x$  squared plus 3.
- Differentiate the outer function to get 5 times the parentheses to the fourth power.
- Multiply by the derivative of the inner function, which is  $4x$ .
- Combine 5 and  $4x$  to get  $20x$  times the quantity  $2x$  squared plus 3 to the fourth power.

**Answer:**  $h'(x) = 20x(2x^2 + 3)^4$

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6. Find the derivative using the chain rule.

$$p(x) = \sqrt{3x^2 - 5}$$

- Rewrite the square root as the inside function raised to the one-half power.
- Differentiate the outer function to get one-half times the parentheses to the negative one-half power.
- Multiply by the derivative of the inner function  $3x$  squared minus 5, which is  $6x$ .
- Simplify by canceling one-half with  $6x$  to get  $3x$  divided by the square root of the inside.

**Answer:**  $p'(x) = \frac{3x}{\sqrt{3x^2 - 5}}$

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7. Find the derivative using the chain rule.

$$q(x) = (x^3 + 2x)^4$$

- Identify the outer function as the parentheses to the fourth power and the inner function as  $x$  cubed plus  $2x$ .
- Differentiate the outer function to get 4 times the parentheses to the third power.
- Multiply by the derivative of the inner function, which is  $3x$  squared plus 2.
- Write the final answer as the product of these factors.

**Answer:**  $q'(x) = 4(x^3 + 2x)^3(3x^2 + 2)$

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8. Find the derivative using the chain rule.

$$r(x) = (7x + 4)^{10}$$

- Identify the outer function as the parentheses to the tenth power and the inner function as  $7x$  plus 4.
- Differentiate the outer function to get 10 times the parentheses to the ninth power.
- Multiply by the derivative of the inner function, which is 7.
- Combine the constants 10 and 7 to get 70 times the quantity  $7x$  plus 4 to the ninth power.

**Answer:**  $r'(x) = 70(7x + 4)^9$

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9. Find the derivative using the chain rule.

$$s(x) = \sqrt[3]{6x - 1}$$

- Rewrite the cube root as the inside function raised to the one-third power.
- Differentiate the outer function to get one-third times the parentheses to the negative two-thirds power.
- Multiply by the derivative of the inner function  $6x$  minus 1, which is 6.
- Simplify by canceling one-third with 6 to get 2 divided by the quantity  $6x$  minus 1 to the two-thirds power.

**Answer:**  $s'(x) = \frac{2}{(6x - 1)^{\frac{2}{3}}}$

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10. Find the derivative using the chain rule.

$$t(x) = (x^2 - 4x + 1)^6$$

- Identify the outer function as the parentheses to the sixth power and the inner function as  $x$  squared minus  $4x$  plus  $1$ .
- Differentiate the outer function to get  $6$  times the parentheses to the fifth power.
- Multiply by the derivative of the inner function, which is  $2x$  minus  $4$ .
- Write the final answer as the product of these factors.

**Answer:**      $t'(x) = 6(x^2 - 4x + 1)^5(2x - 4)$

