Finding Critical Numbers of Functions

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Compute the first derivative of rational and polynomial functions
- Identify values where the derivative equals zero or is undefined
- Determine valid critical numbers by checking the domain of the original function

Find all critical numbers of each function by computing the first derivative and applying both conditions: f'(c) = 0 and f'(c) undefined.

1. Find all critical numbers of the function.

$$f(x) = \frac{x^2}{x^2 - 4}$$

Answer: _____

2. Find all critical numbers of the polynomial.

$$f(x) = x^3 - 3x^2 - 9x + 5$$

Answer: _____

3. Find all critical numbers of the rational function.

$$f(x) = \frac{x}{x^2 + 1}$$

Answer: _____

4. Find all critical numbers of the function.

$$f(x) = x^4 - 8x^2 + 16$$

Answer: _____

5. Find all critical numbers of the radical function.

$$f(x) = \sqrt{x^2 - 9}$$

Answer: _____

6. Find all critical numbers of the function.

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

Answer: _____

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7. Find all critical numbers of the function.

$f(x) = x^{2/3}(x - 5)$

Answer: _____

8. Find all critical numbers of the trigonometric function on [0, 2\pi].

$f(x) = \sin x + \cos x$

Answer: _____

9. Find all critical numbers of the function.

$f(x) = \frac{1}{x^2 - 1}$

Answer: _____

10. Find all critical numbers of the exponential-polynomial function.

$f(x) = xe^{-x}$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students that a candidate value is only a valid critical number if it lies in the domain of the original function f(x).

Solutions

1. Find all critical numbers of the function.

$$f(x) = \frac{x^2}{x^2 - 4}$$

→ *Use the quotient rule to differentiate f(x).*
→ *Simplify the derivative to negative 8x over the quantity x squared minus 4, squared.*
→ *Set the numerator equal to zero to get x equals 0.*
→ *Set the denominator equal to zero to get x equals plus or minus 2.*
→ *Check the domain of f(x); x equals plus or minus 2 are excluded, so only x equals 0 is a valid critical number.*

Answer: $x = 0$

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2. Find all critical numbers of the polynomial.

$$f(x) = x^3 - 3x^2 - 9x + 5$$

→ *Differentiate to obtain f prime of x equals 3x squared minus 6x minus 9.*
→ *Factor as 3 times the quantity x squared minus 2x minus 3.*
→ *Factor further into 3 times x minus 3 times x plus 1.*
→ *Set each factor equal to zero to find x equals 3 and x equals negative 1.*

Answer: $x = -1,\ x = 3$

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3. Find all critical numbers of the rational function.

$$f(x) = \frac{x}{x^2 + 1}$$

→ *Apply the quotient rule to get f prime of x equals the quantity 1 minus x squared over the quantity x squared plus 1, squared.*
→ *Set the numerator 1 minus x squared equal to zero.*
→ *Solve to get x equals plus or minus 1.*
→ *The denominator is never zero, so both values are valid critical numbers.*

Answer: $x = -1,\ x = 1$

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4. Find all critical numbers of the function.

$$f(x) = x^4 - 8x^2 + 16$$

→ *Differentiate to get f prime of x equals 4x cubed minus 16x.*
→ *Factor as 4x times the quantity x squared minus 4.*
→ *Factor further into 4x times x minus 2 times x plus 2.*
→ *Setting each factor equal to zero gives x equals 0, x equals 2, and x equals negative 2.*

Answer: $x = -2,\ x = 0,\ x = 2$

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5. Find all critical numbers of the radical function.

$f(x) = \sqrt{x^2 - 9}$

→ *Differentiate using the chain rule to get f prime of x equals x over the square root of the quantity x squared minus 9.*
→ *Set the numerator equal to zero to get x equals 0, but 0 is not in the domain of f.*
→ *Set the denominator equal to zero to get x equals plus or minus 3.*
→ *Since plus or minus 3 are endpoints of the domain, they are the critical numbers.*

Answer: $x = \pm 3$

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6. Find all critical numbers of the function.

$f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

→ *Apply the quotient rule to obtain f prime of x equals 4x over the quantity x squared plus 1, squared.*
→ *Set the numerator equal to zero to get x equals 0.*
→ *The denominator x squared plus 1 is never zero, so there are no undefined points.*
→ *The only critical number is x equals 0.*

Answer: $x = 0$

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7. Find all critical numbers of the function.

$f(x) = x^{2/3}(x - 5)$

→ *Rewrite f(x) as x to the five-thirds minus 5 times x to the two-thirds.*
→ *Differentiate to get f prime of x equals five-thirds times x to the two-thirds minus ten-thirds times x to the negative one-third.*
→ *Combine over a common denominator to get the quantity 5x minus 10 over 3 times the cube root of x.*
→ *Set the numerator equal to zero to get x equals 2, and the denominator equal to zero to get x equals 0.*
→ *Both values lie in the domain of f, so they are critical numbers.*

Answer: $x = 0,\ x = 2$

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8. Find all critical numbers of the trigonometric function on [0, 2\pi].

$f(x) = \sin x + \cos x$

→ *Differentiate to get f prime of x equals cosine x minus sine x.*
→ *Set f prime of x equal to zero to get cosine x equals sine x.*
→ *Divide both sides by cosine x to get tangent x equals 1.*
→ *On the interval from 0 to 2 pi, the solutions are x equals pi over 4 and x equals 5 pi over 4.*

Answer: $x = \dfrac{\pi}{4},\ x = \dfrac{5\pi}{4}$

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9. Find all critical numbers of the function.

$f(x) = \dfrac{1}{x^2 - 1}$

→ *Differentiate to obtain f prime of x equals negative 2x over the quantity x squared minus 1, squared.*
→ *Set the numerator equal to zero to get x equals 0.*
→ *Set the denominator equal to zero to get x equals plus or minus 1, but these are not in the domain of f.*
→ *Therefore, x equals 0 is the only critical number.*

Answer: $x = 0$

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10. Find all critical numbers of the exponential-polynomial function.

$f(x) = xe^{-x}$

→ *Differentiate using the product rule to get f prime of x equals e to the negative x minus x times e to the negative x.*
→ *Factor out e to the negative x to obtain e to the negative x times the quantity 1 minus x.*
→ *Since e to the negative x is never zero, set 1 minus x equal to zero.*
→ *Solving gives x equals 1 as the only critical number.*

Answer: $x = 1$