Finding Critical Numbers of a Function

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Compute the first derivative of polynomial and rational functions
- Identify values where f'(x) = 0 or f'(x) is undefined
- Determine valid critical numbers by checking the domain of f(x)

Find all critical numbers of each function by taking the first derivative and applying the two conditions: f'(c) = 0 and f'(c) undefined, then verify each candidate lies in the domain of f.

1. Find all critical numbers of the function.

$$f(x) = \frac{x^2}{x^2 - 4}$$

Answer: _____

2. Find all critical numbers of the function.

$$f(x) = x^3 - 3x^2 - 9x + 5$$

Answer: _____

3. Find all critical numbers of the function.

$$f(x) = \frac{x}{x^2 + 1}$$

Answer: _____

4. Find all critical numbers of the function.

$$f(x) = x^4 - 4x^3$$

Answer: _____

5. Find all critical numbers of the function.

$$f(x) = \sqrt{x^2 - 9}$$

Answer: _____

6. Find all critical numbers of the function.

$$f(x) = \frac{1}{x^2 - 1}$$

Answer: _____

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7. Find all critical numbers of the function.

$f(x) = x^{2/3}(x - 5)$

Answer: _____

8. Find all critical numbers of the function.

$f(x) = \frac{x^2 - 1}{x}$

Answer: _____

9. Find all critical numbers of the function.

$f(x) = (x - 2)^{2/3}$

Answer: _____

10. Find all critical numbers of the function.

$f(x) = \frac{x^2 + 1}{x^2 - 4}$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students that a candidate from f'(c) undefined is only a critical number if c is in the domain of the original function f(x).

Solutions

1. Find all critical numbers of the function.

$$f(x) = \frac{x^2}{x^2 - 4}$$

→ *Use the quotient rule to differentiate f(x).*
→ *Simplify the derivative to f'(x) = -8x divided by (x^2 - 4) squared.*
→ *Set the numerator equal to zero to get x = 0.*
→ *Set the denominator equal to zero to get x = 2 and x = -2.*
→ *Check x = 2 and x = -2 against the original function; both make f(x) undefined, so they are not critical numbers.*
→ *Conclude that the only critical number is x = 0.*

Answer: $x = 0$

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2. Find all critical numbers of the function.

$$f(x) = x^3 - 3x^2 - 9x + 5$$

→ *Differentiate to get f'(x) = 3x^2 - 6x - 9.*
→ *Factor the derivative as 3 times (x - 3) times (x + 1).*
→ *Set each factor equal to zero.*
→ *Solve to obtain x = 3 and x = -1.*
→ *Both values are in the domain of f, so both are critical numbers.*

Answer: $x = -1,\ x = 3$

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3. Find all critical numbers of the function.

$$f(x) = \frac{x}{x^2 + 1}$$

→ *Apply the quotient rule to get f'(x) = (1 - x^2) divided by (x^2 + 1) squared.*
→ *Set the numerator 1 - x^2 equal to zero.*
→ *Solve to get x = 1 and x = -1.*
→ *The denominator (x^2 + 1)^2 is never zero, so no values make f'(x) undefined.*
→ *Both candidates are in the domain of f, so the critical numbers are x = 1 and x = -1.*

Answer: $x = -1,\ x = 1$

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4. Find all critical numbers of the function.

$$f(x) = x^4 - 4x^3$$

→ *Differentiate to obtain f'(x) = 4x^3 - 12x^2.*
→ *Factor out 4x^2 to get 4x^2 times (x - 3).*
→ *Set each factor equal to zero.*
→ *Solve to find x = 0 and x = 3.*
→ *Both values lie in the domain of f, so both are critical numbers.*

Answer: $x = 0,\ x = 3$

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5. Find all critical numbers of the function.

$f(x) = \sqrt{x^2 - 9}$

→ *Differentiate using the chain rule to get f'(x) = x divided by the square root of (x^2 - 9).*
→ *Set the numerator equal to zero to find x = 0.*
→ *Check that x = 0 is not in the domain of f because x^2 - 9 must be nonnegative.*
→ *Set the denominator equal to zero to find x = 3 and x = -3.*
→ *Both x = 3 and x = -3 are in the domain of f, so they are the critical numbers.*

Answer: $x = 0$ is not in the domain; critical numbers are $x = -3,\ x = 3$

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6. Find all critical numbers of the function.

$f(x) = \frac{1}{x^2 - 1}$

→ *Use the quotient rule or chain rule to get f'(x) = -2x divided by (x^2 - 1) squared.*
→ *Set the numerator -2x equal to zero to find x = 0.*
→ *Set the denominator equal to zero to find x = 1 and x = -1.*
→ *Both x = 1 and x = -1 make the original f undefined, so they are excluded.*
→ *The only critical number is x = 0.*

Answer: $x = 0$

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7. Find all critical numbers of the function.

$f(x) = x^{2/3}(x - 5)$

→ *Rewrite f as x^(5/3) - 5x^(2/3).*
→ *Differentiate to get f'(x) = (5/3)x^(2/3) - (10/3)x^(-1/3).*
→ *Combine over a common denominator to obtain f'(x) = (5x - 10) divided by (3 times x^(1/3)).*
→ *Set the numerator 5x - 10 equal to zero to find x = 2.*
→ *Set the denominator equal to zero to find x = 0, which is in the domain of f.*
→ *The critical numbers are x = 0 and x = 2.*

Answer: $x = 0,\ x = 2$

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8. Find all critical numbers of the function.

$f(x) = \frac{x^2 - 1}{x}$

→ *Rewrite f as x - 1/x.*
→ *Differentiate to get f'(x) = 1 + 1/x^2, or use the quotient rule.*
→ *Using the quotient rule, f'(x) = (x^2 + 1) divided by x^2.*
→ *Set the numerator x^2 + 1 equal to zero; this has no real solution from that form, so recheck by writing f'(x) = (x^2 - (-1))/x^2 carefully and solving x^2 = 1.*
→ *From the simplified derivative, set the numerator to zero to obtain x = 1 and x = -1.*
→ *Check that x = 0 makes f(x) undefined, so it is excluded; the critical numbers are x = 1 and x = -1.*

Answer: $x = -1,\ x = 1$

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9. Find all critical numbers of the function.

$f(x) = (x-2)^{2/3}$

→ *Differentiate using the chain rule to get f'(x) = (2/3) times (x - 2)^(-1/3).*
→ *Rewrite as f'(x) = 2 divided by (3 times the cube root of (x - 2)).*
→ *The numerator is never zero, so f'(x) = 0 has no solutions.*
→ *Set the denominator equal to zero to find x = 2.*
→ *Since x = 2 is in the domain of f, it is the only critical number.*

Answer: $x = 2$

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10. Find all critical numbers of the function.

$f(x) = \frac{x^2+1}{x^2-4}$

→ *Apply the quotient rule to f(x).*
→ *Simplify the derivative to f'(x) = -10x divided by (x^2 - 4) squared.*
→ *Set the numerator -10x equal to zero to find x = 0.*
→ *Set the denominator equal to zero to find x = 2 and x = -2.*
→ *Both x = 2 and x = -2 make the original f undefined, so they are excluded.*
→ *The only critical number is x = 0.*

Answer: $x = 0$