Using the First Derivative to Analyze Function Behavior

Calculus Worksheet · Grade 11-12

Name: ______________________________ Date: _______________ Score: / 10

Learning Objectives

- Determine intervals where a function is increasing or decreasing using the first derivative
- Locate local maximum and local minimum values using the First Derivative Test
- Interpret the sign of f'(x) to describe function behavior without graphing

For each function, find the critical numbers, determine the intervals of increase and decrease, and identify all local extrema using the First Derivative Test.

1. Find the intervals of increase and decrease and identify all local extrema.

$$f(x) = x^2 - 6x + 5$$

Answer: ________________________________

2. Find the intervals of increase and decrease and identify all local extrema.

$$f(x) = -x^2 + 4x + 1$$

Answer: ________________________________

3. Find the critical numbers and classify each as a local maximum, local minimum, or neither.

$$f(x) = x^3 - 3x^2 - 9x + 5$$

Answer: ________________________________

4. Determine where the function is increasing and decreasing.

$$f(x) = 2x^3 - 9x^2 + 12x - 4$$

Answer: ________________________________

5. Find the local extrema using the First Derivative Test.

$$f(x) = x^4 - 4x^3$$

Answer: ________________________________

6. Find the intervals of increase and decrease.

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 7$$

Answer: ________________________________

7. Identify all local maxima and minima.

$$f(x) = x^3 - 12x + 1$$

Answer: ________________________________

8. Find the local extrema of the function.

$$f(x) = x + \frac{4}{x}$$

Answer: ______________________________________

9. Determine the intervals on which the function is increasing or decreasing.

$$f(x) = x^4 - 8x^2 + 3$$

Answer: ______________________________________

10. Find all local maxima and minima.

$$f(x) = 3x^5 - 5x^3$$

Answer: ______________________________________

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ANSWER KEY & SOLUTIONS

Encourage students to build a sign chart for f′(x) and verify sign changes at each critical number to classify extrema.

Solutions

1. Find the intervals of increase and decrease and identify all local extrema.

$f(x) = x^2 - 6x + 5$

→ *Compute the first derivative: f prime of x equals 2x minus 6.*
→ *Set the derivative equal to zero to find the critical number: 2x minus 6 equals 0, so x equals 3.*
→ *Test values on either side of 3: f prime of 2 is negative, f prime of 4 is positive.*
→ *The derivative changes from negative to positive at x equals 3, so there is a local minimum at x equals 3.*
→ *Evaluate f at 3 to get the minimum value of negative 4.*

Answer: $\text{Decreasing on } (-\infty, 3),\ \text{Increasing on } (3, \infty),\ \text{Local min at } x = 3,\ f(3) = -4$

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2. Find the intervals of increase and decrease and identify all local extrema.

$f(x) = -x^2 + 4x + 1$

→ *Differentiate to get f prime of x equals negative 2x plus 4.*
→ *Set the derivative equal to zero: negative 2x plus 4 equals 0, so x equals 2.*
→ *Pick test values: f prime of 1 is positive, f prime of 3 is negative.*
→ *Since the derivative changes from positive to negative, there is a local maximum at x equals 2.*
→ *Evaluate the original function at 2 to obtain the local maximum value 5.*

Answer: $\text{Increasing on } (-\infty, 2),\ \text{Decreasing on } (2, \infty),\ \text{Local max at } x = 2,\ f(2) = 5$

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3. Find the critical numbers and classify each as a local maximum, local minimum, or neither.

$f(x) = x^3 - 3x^2 - 9x + 5$

→ *Differentiate to get f prime of x equals 3x squared minus 6x minus 9.*
→ *Factor as 3 times the quantity x minus 3 times x plus 1 and set equal to zero.*
→ *The critical numbers are x equals negative 1 and x equals 3.*
→ *Test signs on each interval: positive on the left of negative 1, negative between negative 1 and 3, positive after 3.*
→ *The derivative changes positive to negative at negative 1, giving a local maximum, and negative to positive at 3, giving a local minimum.*
→ *Plug each critical number into f to find the function values 10 and negative 22.*

Answer: $\text{Local max at } x = -1,\ f(-1) = 10;\ \text{Local min at } x = 3,\ f(3) = -22$

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4. Determine where the function is increasing and decreasing.

$f(x) = 2x^3 - 9x^2 + 12x - 4$

→ *Compute f prime of x equals 6x squared minus 18x plus 12.*
→ *Factor out 6 to get 6 times the quantity x minus 1 times x minus 2.*
→ *The critical numbers are x equals 1 and x equals 2.*
→ *Test the sign of the derivative in each interval: positive on the left of 1, negative between 1 and 2, positive after 2.*
→ *Conclude the function increases on the outer intervals and decreases between 1 and 2.*

Answer: $\text{Increasing on } (-\infty, 1) \cup (2, \infty),\ \text{Decreasing on } (1, 2)$

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5. Find the local extrema using the First Derivative Test.

$f(x) = x^4 - 4x^3$

→ *Differentiate to obtain f prime of x equals 4x cubed minus 12x squared.*
→ *Factor as 4x squared times the quantity x minus 3 and set equal to zero.*
→ *The critical numbers are x equals 0 and x equals 3.*
→ *Test the sign of the derivative: negative on both sides of 0, negative just before 3, and positive after 3.*
→ *At x equals 0 there is no sign change so it is neither a maximum nor minimum, while at x equals 3 the derivative changes from negative to positive giving a local minimum.*
→ *Evaluate f at 3 to obtain the local minimum value negative 27.*

Answer: $\text{Local min at } x = 3,\ f(3) = -27;\ \text{No extremum at } x = 0$

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6. Find the intervals of increase and decrease.

$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 7$

→ *Differentiate to get f prime of x equals x squared minus 2x minus 3.*
→ *Factor the derivative as the quantity x minus 3 times x plus 1.*
→ *Set each factor to zero to find critical numbers x equals negative 1 and x equals 3.*
→ *Test signs: positive when x is less than negative 1, negative between negative 1 and 3, positive when x is greater than 3.*
→ *State that the function increases on the outer intervals and decreases on the middle interval.*

Answer: $\text{Increasing on } (-\infty, -1) \cup (3, \infty),\ \text{Decreasing on } (-1, 3)$

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7. Identify all local maxima and minima.

$f(x) = x^3 - 12x + 1$

→ *Take the derivative: f prime of x equals 3x squared minus 12.*
→ *Set the derivative equal to zero and solve to get x equals negative 2 and x equals 2.*
→ *Test the sign of f prime in each region: positive on the far left, negative in the middle, positive on the far right.*
→ *Since the derivative changes from positive to negative at negative 2, there is a local maximum; at 2 it changes from negative to positive giving a local minimum.*
→ *Evaluate the function at each critical number to find the corresponding values 17 and negative 15.*

Answer: $\text{Local max at } x = -2,\ f(-2) = 17;\ \text{Local min at } x = 2,\ f(2) = -15$

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8. Find the local extrema of the function.

$f(x) = x + \frac{4}{x}$

→ *Rewrite the function and differentiate to get f prime of x equals 1 minus 4 divided by x squared.*
→ *Set the derivative equal to zero: 1 minus 4 over x squared equals 0, giving x squared equals 4.*
→ *Solve for x to get critical numbers x equals negative 2 and x equals 2; note x equals 0 is not in the domain.*
→ *Test the sign of the derivative in each interval around the critical numbers and the excluded value 0.*
→ *At x equals negative 2 the derivative changes from positive to negative so there is a local maximum, and at x equals 2 it changes from negative to positive giving a local minimum.*
→ *Plug each critical number into f to obtain the values negative 4 and 4.*

Answer: $\text{Local max at } x = -2,\ f(-2) = -4;\ \text{Local min at } x = 2,\ f(2) = 4$

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9. Determine the intervals on which the function is increasing or decreasing.

$f(x) = x^4 - 8x^2 + 3$

→ *Differentiate to get f prime of x equals 4x cubed minus 16x.*

→ *Factor as 4x times the quantity x squared minus 4, which equals 4x times x minus 2 times x plus 2.*

→ *Solve to find critical numbers x equals negative 2, x equals 0, and x equals 2.*

→ *Build a sign chart and test the sign of f prime in each of the four intervals.*

→ *Conclude the function decreases on the outer-left and inner-right intervals, and increases on the inner-left and outer-right intervals.*

Answer: $\text{Decreasing on } (-\infty, -2) \cup (0, 2),\ \text{Increasing on } (-2, 0) \cup (2, \infty)$

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10. Find all local maxima and minima.

$f(x) = 3x^5 - 5x^3$

→ *Differentiate to obtain f prime of x equals 15x to the fourth minus 15x squared.*

→ *Factor as 15x squared times the quantity x minus 1 times x plus 1.*

→ *The critical numbers are x equals negative 1, x equals 0, and x equals 1.*

→ *Test the sign of the derivative in each interval determined by these numbers.*

→ *At negative 1 the sign changes from positive to negative giving a local maximum, at 0 there is no sign change so no extremum, and at 1 the sign changes from negative to positive giving a local minimum.*

→ *Evaluate the function at the critical numbers negative 1 and 1 to obtain the values 2 and negative 2.*

Answer: $\text{Local max at } x = -1,\ f(-1) = 2;\ \text{Local min at } x = 1,\ f(1) = -2;\ \text{No extremum at } x = 0$