First and Second Derivative Test Using the TI-84 Calculator

Calculus Worksheet · Grade 11-12

Name: ______________________ Date: ______________ Score: / 10

Learning Objectives

- Use the TI-84 calculator to compute derivatives numerically
- Find critical numbers by locating zeros of the first derivative
- Apply the first and second derivative tests to classify local extrema

For each problem, use the first and second derivative tests to find critical numbers and classify them as local maxima or minima.

1. Find the critical numbers of the function.

$$f(x) = x^3 - 3x^2 + x - 2$$

Answer: ______________________

2. Classify the critical numbers using the first derivative test.

$$f(x) = x^3 - 3x^2 + x - 2$$

Answer: ______________________

3. Use the second derivative test to classify the critical points.

$$f(x) = x^3 - 3x^2 + x - 2$$

Answer: ______________________

4. Find the critical numbers of the cubic function.

$$f(x) = x^3 - 6x^2 + 9x + 1$$

Answer: ______________________

5. Apply the first derivative test to classify the critical points.

$$f(x) = x^3 - 6x^2 + 9x + 1$$

Answer: ______________________

6. Find the critical number of the quadratic function.

$$f(x) = 2x^2 - 8x + 5$$

Answer: ______________________

7. Use the second derivative test to classify the critical point.

$$f(x) = 2x^2 - 8x + 5$$

Answer: ______________________

8. Find the critical numbers of the quartic function.

$f(x) = x^4 - 4x^3$

Answer: ______________________________

9. Classify the critical numbers using the second derivative test.

$f(x) = x^4 - 4x^3$

Answer: ______________________________

10. Determine the intervals where the function is increasing and decreasing.

$f(x) = x^3 - 3x^2 + x - 2$

Answer: ______________________________

Encourage students to verify graphical results from the TI-84 with analytical derivative computations when possible.

Solutions

1. Find the critical numbers of the function.

$f(x) = x^3 - 3x^2 + x - 2$

→ *Compute the first derivative of f of x.*
→ *Set the first derivative equal to zero.*
→ *Use the calculator's intersect feature to find where the derivative crosses the x-axis.*
→ *Record both x-values as the critical numbers.*

Answer: $x \approx 0.184,\ x \approx 1.816$

2. Classify the critical numbers using the first derivative test.

$f(x) = x^3 - 3x^2 + x - 2$

→ *Build a sign chart for f prime of x using the two critical numbers.*
→ *Check the sign of the derivative to the left of 0.184 and find it is positive.*
→ *Check the sign of the derivative between the critical numbers and find it is negative.*
→ *Check the sign of the derivative to the right of 1.816 and find it is positive.*
→ *Conclude that 0.184 gives a local maximum and 1.816 gives a local minimum.*

Answer: $\text{Local max at } x \approx 0.184,\ \text{Local min at } x \approx 1.816$

3. Use the second derivative test to classify the critical points.

$f(x) = x^3 - 3x^2 + x - 2$

→ *Compute the second derivative of f of x.*
→ *Evaluate the second derivative at each critical number using the calculator.*
→ *If the second derivative is negative the point is a local maximum.*
→ *If the second derivative is positive the point is a local minimum.*

Answer: $f''(0.184) < 0 \Rightarrow \text{local max};\ f''(1.816) > 0 \Rightarrow \text{local min}$

4. Find the critical numbers of the cubic function.

$f(x) = x^3 - 6x^2 + 9x + 1$

→ *Enter the function into Y1 on the TI-84.*
→ *Use the derivative function to graph the first derivative.*
→ *Use the intersect feature with y equals zero to find the zeros.*
→ *Read off the two critical numbers.*

Answer: $x = 1,\ x = 3$

5. Apply the first derivative test to classify the critical points.

$f(x) = x^3 - 6x^2 + 9x + 1$

→ *Make a sign chart using x equals 1 and x equals 3.*
→ *Test the sign of the derivative on each interval.*
→ *Mark intervals where the derivative is positive as increasing.*
→ *Mark intervals where the derivative is negative as decreasing.*
→ *Identify the local max and local min from the sign changes.*

Answer: $\text{Local max at } x = 1, \text{ Local min at } x = 3$

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6. Find the critical number of the quadratic function.

$f(x) = 2x^2 - 8x + 5$

→ *Enter the function into the calculator.*
→ *Compute the first derivative numerically.*
→ *Graph the derivative and use intersect with the x-axis.*
→ *Identify x equals 2 as the only critical number.*

Answer: $x = 2$

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7. Use the second derivative test to classify the critical point.

$f(x) = 2x^2 - 8x + 5$

→ *Compute the second derivative which is a constant.*
→ *Evaluate the second derivative at x equals 2.*
→ *Since the second derivative is positive the critical point is a local minimum.*

Answer: $x = 2 \text{ is a local minimum}$

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8. Find the critical numbers of the quartic function.

$f(x) = x^4 - 4x^3$

→ *Graph the function and its derivative on the TI-84.*
→ *Adjust the window to clearly see where the derivative crosses zero.*
→ *Use the intersect tool to find the zeros of the derivative.*
→ *Record both critical numbers.*

Answer: $x = 0, \; x = 3$

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9. Classify the critical numbers using the second derivative test.

$f(x) = x^4 - 4x^3$

→ *Compute the second derivative of the function.*
→ *Evaluate the second derivative at each critical number.*
→ *At x equals 0 the second derivative is zero so the test is inconclusive.*
→ *At x equals 3 the second derivative is positive so the point is a local minimum.*
→ *Use the first derivative test at x equals 0 to confirm it is not an extremum.*

Answer: $x = 0 \text{ is inconclusive (saddle/inflection); } x = 3 \text{ is a local minimum}$

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10. Determine the intervals where the function is increasing and decreasing.

$f(x) = x^3 - 3x^2 + x - 2$

$\rightarrow$ *Use the previously found critical numbers from the derivative graph.*
$\rightarrow$ *Examine the sign of the derivative on each interval.*
$\rightarrow$ *Where the derivative is positive the function is increasing.*
$\rightarrow$ *Where the derivative is negative the function is decreasing.*
$\rightarrow$ *Write the intervals in interval notation.*

Answer: Increasing on $(-\infty, 0.184) \cup (1.816, \infty)$; Decreasing on $(0.184, 1.816)$