

Implicit Differentiation

AP Calculus Worksheet · Grade 11–12

Name: _____

Date: _____

Learning Objectives

- Distinguish between derivatives in terms of the same variable versus different variables
- Apply implicit differentiation to find dy/dx for equations involving x and y
- Solve for dy/dx algebraically after differentiating both sides of an equation

Problems

1. Find the derivative of the function with respect to x using the power rule. Write your answer using prime notation.

$$f(x) = x^7$$

2. Find the derivative of y to the seventh power with respect to x using implicit differentiation. Keep the dy/dx notation in your answer.

$$y^7$$

3. Differentiate each term with respect to x . Keep the dy/dx notation for the y term.

$$x^4 + y^4$$

4. Use implicit differentiation to differentiate both sides of the equation with respect to x , then solve for dy/dx .

$$5y^9 = 1$$

5. Find dy/dx by differentiating both sides of the equation with respect to x using implicit differentiation.

$$x^2 + y^2 = 25$$

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6. Differentiate both sides of the equation with respect to x and solve for dy/dx .

$$x^3 + y^3 = 8$$

7. Use implicit differentiation to find dy/dx for the equation below. Differentiate both sides with respect to x .

$$x^2y + y^3 = 10$$

8. Apply implicit differentiation to find dy/dx . Differentiate both sides with respect to x , then isolate dy/dx .

$$4x^2 - 3y^2 = 7x$$

9. Use implicit differentiation to find dy/dx for the equation below, which involves a product of x and y on the left side.

$$x^3y^2 - 5x + 2y = 0$$

10. Find the equation of the tangent line to the curve at the given point by first using implicit differentiation to find dy/dx , then substituting the point to find the slope.

$$x^2 + xy + y^2 = 7 \text{ at the point } (1, 2)$$

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Implicit Differentiation — Answer Key

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Answer Key

1. Answer: $f'(x) = 7x^6$

- Apply the power rule: bring down the exponent and reduce it by 1.
- $f'(x) = 7x^{(7-1)} = 7x^6$

2. Answer: $7y^6 \cdot dy/dx$

- Apply the power rule to y^7 , treating y as a function of x .
- The derivative is $7y^6$, but because we are differentiating y with respect to x , we must multiply by dy/dx .
- Final answer: $7y^6 \cdot dy/dx$

3. Answer: $4x^3 + 4y^3 \cdot dy/dx$

- Differentiate x^4 with respect to x : $4x^3$ (the $dx/dx = 1$ term drops out).
- Differentiate y^4 with respect to x using implicit differentiation: $4y^3 \cdot dy/dx$.
- Combine: $4x^3 + 4y^3 \cdot dy/dx$

4. Answer: $dy/dx = 0$

- Differentiate the left side: $d/dx(5y^9) = 45y^8 \cdot dy/dx$.
- Differentiate the right side: $d/dx(1) = 0$.
- Set them equal: $45y^8 \cdot dy/dx = 0$.
- Divide both sides by $45y^8$: $dy/dx = 0$.

5. Answer: $dy/dx = -x/y$

- Differentiate both sides with respect to x : $2x + 2y \cdot dy/dx = 0$.
- Isolate the dy/dx term: $2y \cdot dy/dx = -2x$.
- Divide both sides by $2y$: $dy/dx = -x/y$.

6. Answer: $dy/dx = -x^2/y^2$

- Differentiate both sides: $3x^2 + 3y^2 \cdot dy/dx = 0$.
- Isolate dy/dx : $3y^2 \cdot dy/dx = -3x^2$.
- Divide both sides by $3y^2$: $dy/dx = -x^2/y^2$.

7. Answer: $dy/dx = -2xy / (x^2 + 3y^2)$

- Differentiate $x^2 \cdot y$ using the product rule: $2x \cdot y + x^2 \cdot (dy/dx)$.
- Differentiate y^3 : $3y^2 \cdot dy/dx$.
- Differentiate 10 : 0 .
- Equation: $2xy + x^2 \cdot (dy/dx) + 3y^2 \cdot (dy/dx) = 0$.
- Factor dy/dx : $dy/dx(x^2 + 3y^2) = -2xy$.
- Solve: $dy/dx = -2xy / (x^2 + 3y^2)$.

8. Answer: $dy/dx = (8x - 7) / (6y)$

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- Differentiate both sides: $8x - 6y \cdot dy/dx = 7$.
- Isolate the dy/dx term: $-6y \cdot dy/dx = 7 - 8x$.
- Divide both sides by $-6y$: $dy/dx = (8x - 7) / (6y)$.

9. Answer: $dy/dx = (5 - 3x^2 \cdot y^2) / (2x^3 \cdot y + 2)$

- Differentiate $x^3 \cdot y^2$ using the product rule: $3x^2 \cdot y^2 + x^3 \cdot (2y) \cdot (dy/dx)$.
- Differentiate $-5x$: -5 .
- Differentiate $2y$: $2 \cdot (dy/dx)$.
- Full equation: $3x^2 \cdot y^2 + 2x^3 \cdot y \cdot (dy/dx) - 5 + 2 \cdot (dy/dx) = 0$.
- Group dy/dx terms: $dy/dx(2x^3 \cdot y + 2) = 5 - 3x^2 \cdot y^2$.
- Solve: $dy/dx = (5 - 3x^2 \cdot y^2) / (2x^3 \cdot y + 2)$.

10. Answer: $y = -(4/5)(x - 1) + 2$, or $y = -4x/5 + 14/5$

- Differentiate both sides with respect to x : $2x + (y + x \cdot dy/dx) + 2y \cdot (dy/dx) = 0$.
- Expand: $2x + y + x \cdot (dy/dx) + 2y \cdot (dy/dx) = 0$.
- Factor dy/dx : $dy/dx(x + 2y) = -(2x + y)$.
- Solve: $dy/dx = -(2x + y) / (x + 2y)$.
- Substitute $(1, 2)$: $dy/dx = -(2 \cdot 1 + 2) / (1 + 2 \cdot 2) = -4/5$.
- Use point-slope form: $y - 2 = -4/5 \cdot (x - 1)$, so $y = -4x/5 + 14/5$.

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