# Derivatives of Inverse Trigonometric Functions

Calculus Worksheet · Grade 11-12

Name: ________________________________ Date: ________________ Score: / 10

**Learning Objectives**

- Apply derivative formulas for inverse trigonometric functions
- Use the chain rule with inverse trig functions
- Combine product rule and inverse trig derivatives to differentiate composite expressions

*Find the derivative of each function using the inverse trigonometric derivative formulas, simplifying your answer completely.*

### 1. Find the derivative of the function.

$y = \sqrt{\tan^{-1} x}$

Answer: ___________________________________

### 2. Find the derivative of the function.

$f(x) = \sin^{-1}(5x)$

Answer: ___________________________________

### 3. Find the derivative of the function using the product rule.

$f(x) = \sqrt{x^2 - 1} \cdot \sec^{-1} x$

Answer: ___________________________________

### 4. Find the derivative of the function.

$y = \cos^{-1}(3x^2)$

Answer: ___________________________________

### 5. Find the derivative of the function.

$y = \tan^{-1}(x^3)$

Answer: ___________________________________

### 6. Find the derivative of the function.

$f(x) = x\sin^{-1} x$

Answer: ___________________________________

**7. Find the derivative of the function.**

$y = \cot^{-1}(2x)$

Answer: ________________________________

**8. Find the derivative of the function.**

$y = \sqrt{\sin^{-1} x}$

Answer: ________________________________

**9. Find the derivative of the function.**

$f(x) = \tan^{-1}(e^x)$

Answer: ________________________________

**10. Find the derivative of the function.**

$y = \csc^{-1}(x^2)$

Answer: ________________________________

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ANSWER KEY & SOLUTIONS

*Remind students to rewrite radicals as fractional exponents before differentiating and to memorize the six inverse trig derivative formulas.*

## Solutions

**1.** Find the derivative of the function.

$y = \sqrt{\tan^{-1} x}$

→ *Rewrite the square root as the one-half power of inverse tangent of x.*
→ *Apply the chain rule with u equal to inverse tangent of x.*
→ *Multiply one-half times u to the negative one-half by the derivative of inverse tangent of x, which is one over one plus x squared.*
→ *Move the negative exponent to the denominator and simplify.*

**Answer:** $$y' = \frac{1}{2(1+x^2)\sqrt{\tan^{-1} x}}$$

**2.** Find the derivative of the function.

$f(x) = \sin^{-1}(5x)$

→ *Identify u as 5x so that du/dx equals 5.*
→ *Apply the inverse sine derivative formula: one over the square root of one minus u squared.*
→ *Substitute 5x for u and multiply by the derivative of u, which is 5.*
→ *Simplify the squared term in the radicand to get 25x squared.*

**Answer:** $$f'(x) = \frac{5}{\sqrt{1-25x^2}}$$

**3.** Find the derivative of the function using the product rule.

$f(x) = \sqrt{x^2-1} \cdot \sec^{-1} x$

→ *Rewrite the radical as x squared minus one to the one-half power.*
→ *Apply the product rule to the two factors.*
→ *Differentiate x squared minus one to the one-half using the chain rule.*
→ *Differentiate inverse secant of x using its formula and combine the two terms.*

**Answer:** $$f'(x) = \frac{x\sec^{-1} x}{\sqrt{x^2-1}} + \frac{\sqrt{x^2-1}}{|x|\sqrt{x^2-1}}$$

**4.** Find the derivative of the function.

$y = \cos^{-1}(3x^2)$

→ *Let u equal 3x squared so that du/dx equals 6x.*
→ *Apply the inverse cosine formula: negative one over the square root of one minus u squared.*
→ *Substitute u into the formula and multiply by du/dx.*
→ *Simplify the radicand to one minus 9x to the fourth.*

**Answer:** $$y' = \frac{-6x}{\sqrt{1-9x^4}}$$

**5.** Find the derivative of the function.

$y = \tan^{-1}(x^3)$

→ *Let u equal x cubed so that du/dx equals 3x squared.*
→ *Apply the inverse tangent formula: one over one plus u squared.*
→ *Substitute u and multiply by du/dx.*
→ *Simplify the denominator to one plus x to the sixth.*

**Answer:** $y' = \dfrac{3x^2}{1 + x^6}$

**6.** Find the derivative of the function.

$f(x) = x\sin^{-1} x$

→ *Apply the product rule to x times inverse sine of x.*
→ *Differentiate x to get 1 and keep inverse sine of x.*
→ *Differentiate inverse sine of x using its formula.*
→ *Combine the two terms to get the final derivative.*

**Answer:** $f'(x) = \sin^{-1} x + \dfrac{x}{\sqrt{1 - x^2}}$

**7.** Find the derivative of the function.

$y = \cot^{-1}(2x)$

→ *Let u equal 2x so that du/dx equals 2.*
→ *Apply the inverse cotangent formula: negative one over one plus u squared.*
→ *Substitute u and multiply by du/dx.*
→ *Simplify the denominator to one plus 4x squared.*

**Answer:** $y' = \dfrac{-2}{1 + 4x^2}$

**8.** Find the derivative of the function.

$y = \sqrt{\sin^{-1} x}$

→ *Rewrite the square root as inverse sine of x to the one-half power.*
→ *Apply the chain rule with u equal to inverse sine of x.*
→ *Multiply one-half times u to the negative one-half by the derivative of inverse sine.*
→ *Move the negative exponent to the denominator and simplify.*

**Answer:** $y' = \dfrac{1}{2\sqrt{\sin^{-1} x} \cdot \sqrt{1 - x^2}}$

**9.** Find the derivative of the function.

$f(x) = \tan^{-1}(e^x)$

→ *Let u equal e to the x so that du/dx equals e to the x.*
→ *Apply the inverse tangent formula: one over one plus u squared.*
→ *Substitute u and multiply by du/dx.*
→ *Simplify the denominator to one plus e to the 2x.*

**Answer:** $f'(x) = \dfrac{e^x}{1 + e^{2x}}$

**10.** Find the derivative of the function.

$y = \csc^{-1}(x^2)$

→ *Let u equal x squared so that du/dx equals 2x.*
→ *Apply the inverse cosecant formula: negative one over the absolute value of u times the square root of u squared minus one.*
→ *Substitute u and multiply by du/dx.*
→ *Simplify by canceling an x to obtain the final answer.*

**Answer:** $y' = \dfrac{-2}{|x|\sqrt{x^4 - 1}}$