# Derivative Using the Limit Definition

Calculus Worksheet · Grade 11-12

Name: ______________________________ Date: _______________ Score: / 10

**Learning Objectives**

- Apply the limit definition of the derivative to find f'(x)
- Simplify difference quotients by expanding and combining like terms
- Evaluate limits as h approaches zero to obtain the derivative

*Use the limit definition of the derivative, f'(x) = lim as h approaches 0 of [f(x+h) - f(x)]/h, to find the derivative of each function.*

**1. Find the derivative using the limit definition.**

$f(x) = 5x + 3$

Answer: ________________________________

**2. Find the derivative using the limit definition.**

$f(x) = 3x^2 - 2x$

Answer: ________________________________

**3. Find the derivative using the limit definition.**

$f(x) = 7x - 4$

Answer: ________________________________

**4. Find the derivative using the limit definition.**

$f(x) = x^2$

Answer: ________________________________

**5. Find the derivative using the limit definition.**

$f(x) = x^2 + 4x$

Answer: ________________________________

**6. Find the derivative using the limit definition.**

$f(x) = 2x^2 - 5x + 1$

Answer: ________________________________

**7. Find the derivative using the limit definition.**

$f(x) = x^3$

Answer: ________________________________

**8. Find the derivative using the limit definition.**

$$f(x) = \frac{1}{x}$$

Answer: ______________________________

**9. Find the derivative using the limit definition.**

$$f(x) = \sqrt{x}$$

Answer: ______________________________

**10. Find the derivative using the limit definition.**

$$f(x) = 4x^2 + 3$$

Answer: ______________________________

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**ANSWER KEY & SOLUTIONS**

*Encourage students to carefully expand f(x+h) and distribute negative signs before simplifying the difference quotient.*

## Solutions

**1.** Find the derivative using the limit definition.

$f(x) = 5x + 3$

→ *Compute f(x+h) by replacing x with x+h, giving 5(x+h) + 3.*
→ *Form the difference quotient: [5(x+h) + 3 - (5x + 3)] / h.*
→ *Distribute and simplify the numerator to get 5h/h.*
→ *Cancel h and take the limit as h approaches 0 to get 5.*

**Answer:** $f'(x) = 5$

**2.** Find the derivative using the limit definition.

$f(x) = 3x^2 - 2x$

→ *Compute f(x+h) = 3(x+h)^2 - 2(x+h) = 3x^2 + 6xh + 3h^2 - 2x - 2h.*
→ *Subtract f(x) to get 6xh + 3h^2 - 2h in the numerator.*
→ *Factor h from the numerator and divide by h to obtain 6x + 3h - 2.*
→ *Take the limit as h approaches 0 to get 6x - 2.*

**Answer:** $f'(x) = 6x - 2$

**3.** Find the derivative using the limit definition.

$f(x) = 7x - 4$

→ *Compute f(x+h) = 7(x+h) - 4 = 7x + 7h - 4.*
→ *Form the difference quotient: (7x + 7h - 4 - 7x + 4) / h = 7h/h.*
→ *Simplify the expression to 7.*
→ *Take the limit as h approaches 0 to get 7.*

**Answer:** $f'(x) = 7$

**4.** Find the derivative using the limit definition.

$f(x) = x^2$

→ *Compute f(x+h) = (x+h)^2 = x^2 + 2xh + h^2.*
→ *Subtract f(x) = x^2 to get 2xh + h^2 in the numerator.*
→ *Divide by h to obtain 2x + h.*
→ *Take the limit as h approaches 0 to get 2x.*

**Answer:** $f'(x) = 2x$

**5.** Find the derivative using the limit definition.

$f(x) = x^2 + 4x$

→ *Compute f(x+h) = (x+h)^2 + 4(x+h) = x^2 + 2xh + h^2 + 4x + 4h.*
→ *Subtract f(x) = x^2 + 4x to get 2xh + h^2 + 4h.*
→ *Factor h and divide to obtain 2x + h + 4.*
→ *Take the limit as h approaches 0 to get 2x + 4.*

**Answer:** $f'(x) = 2x + 4$

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**6.** Find the derivative using the limit definition.

$f(x) = 2x^2 - 5x + 1$

→ *Compute f(x+h) = 2(x+h)^2 - 5(x+h) + 1 = 2x^2 + 4xh + 2h^2 - 5x - 5h + 1.*
→ *Subtract f(x) to leave 4xh + 2h^2 - 5h in the numerator.*
→ *Factor h and divide by h to obtain 4x + 2h - 5.*
→ *Take the limit as h approaches 0 to get 4x - 5.*

**Answer:** $f'(x) = 4x - 5$

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**7.** Find the derivative using the limit definition.

$f(x) = x^3$

→ *Compute f(x+h) = (x+h)^3 = x^3 + 3x^2 h + 3xh^2 + h^3.*
→ *Subtract f(x) = x^3 to get 3x^2 h + 3xh^2 + h^3.*
→ *Factor h and divide by h to obtain 3x^2 + 3xh + h^2.*
→ *Take the limit as h approaches 0 to get 3x^2.*

**Answer:** $f'(x) = 3x^2$

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**8.** Find the derivative using the limit definition.

$f(x) = \frac{1}{x}$

→ *Compute f(x+h) = 1/(x+h).*
→ *Form the difference quotient: [1/(x+h) - 1/x] / h = [-h / (x(x+h))] / h.*
→ *Simplify by canceling h to get -1 / (x(x+h)).*
→ *Take the limit as h approaches 0 to obtain -1/x^2.*

**Answer:** $f'(x) = -\frac{1}{x^2}$

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**9.** Find the derivative using the limit definition.

$f(x) = \sqrt{x}$

→ *Form the difference quotient: [sqrt(x+h) - sqrt(x)] / h.*
→ *Multiply numerator and denominator by the conjugate sqrt(x+h) + sqrt(x).*
→ *Simplify the numerator to h, giving h / [h(sqrt(x+h) + sqrt(x))].*
→ *Cancel h and take the limit as h approaches 0 to get 1/(2 sqrt(x)).*

**Answer:** $f'(x) = \frac{1}{2\sqrt{x}}$

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**10.** Find the derivative using the limit definition.

$f(x) = 4x^2 + 3$

→ *Compute f(x+h) = 4(x+h)^2 + 3 = 4x^2 + 8xh + 4h^2 + 3.*
→ *Subtract f(x) = 4x^2 + 3 to get 8xh + 4h^2.*
→ *Factor h and divide by h to obtain 8x + 4h.*
→ *Take the limit as h approaches 0 to get 8x.*

**Answer:** $f'(x) = 8x$