# Derivatives of Logarithmic Functions

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

**Learning Objectives**

- Apply the derivative formulas for logarithmic and natural logarithmic functions
- Use the chain rule when differentiating composite logarithmic expressions
- Simplify derivatives using logarithm expansion properties

*Find the derivative of each logarithmic function and simplify your answer completely.*

**1. Find the derivative of the function.**

$f(x) = \log_3(5x)$

Answer: _____

**2. Find the derivative of the function.**

$y = \log(2 + \sin x)$

Answer: _____

**3. Find the derivative of the function.**

$f(x) = \ln(x^2 + 1)$

Answer: _____

**4. Find the derivative of the function.**

$y = \ln(\sqrt{x})$

Answer: _____

**5. Find the derivative of the function.**

$f(x) = \log_2(x^3 - 4x)$

Answer: _____

**6. Find the derivative of the function.**

$y = \ln\left(\frac{x}{x+1}\right)$

Answer: _____

**7. Find the derivative of the function.**

$f(x) = \ln(\cos x)$

Answer: _____



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**8. Find the derivative of the function.**

$y = \log_5(4x^2 + 7)$

Answer: _____

**9. Find the derivative of the function.**

$f(x) = \ln(x^2 \cdot e^x)$

Answer: _____

**10. Find the derivative of the function.**

$y = \log_7(\tan x)$

Answer: _____

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**ANSWER KEY & SOLUTIONS**

*Remind students that \log x without a base implies base 10, and emphasize identifying u and du before applying the formula.*

## Solutions

**1.** Find the derivative of the function.

$f(x) = \log_3(5x)$

→ *Identify u equals 5x and du equals 5.*
→ *Apply the formula derivative of log base a of u equals du over u times ln a.*
→ *Substitute to get 5 over 5x times ln 3.*
→ *Cancel the 5 in numerator and denominator to get 1 over x ln 3.*

**Answer:** $f'(x) = \dfrac{1}{x \ln 3}$

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**2.** Find the derivative of the function.

$y = \log(2 + \sin x)$

→ *Identify u equals 2 plus sine x and du equals cosine x.*
→ *Recognize that log with no base means base 10, so use ln 10.*
→ *Apply the logarithm derivative formula.*
→ *Write the simplified result as cosine x over the quantity 2 plus sine x times ln 10.*

**Answer:** $\dfrac{dy}{dx} = \dfrac{\cos x}{(2 + \sin x)\ln 10}$

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**3.** Find the derivative of the function.

$f(x) = \ln(x^2 + 1)$

→ *Identify u equals x squared plus 1 and du equals 2x.*
→ *Apply the natural log derivative formula du over u.*
→ *Write the final answer as 2x over x squared plus 1.*

**Answer:** $f'(x) = \dfrac{2x}{x^2 + 1}$

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**4.** Find the derivative of the function.

$y = \ln(\sqrt{x})$

→ *Rewrite ln of square root of x as one half times ln x using the exponent expansion rule.*
→ *Differentiate one half times ln x to get one half times 1 over x.*
→ *Simplify to 1 over 2x.*

**Answer:** $\dfrac{dy}{dx} = \dfrac{1}{2x}$

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**5.** Find the derivative of the function.

$f(x) = \log_2(x^3 - 4x)$

→ *Identify u equals x cubed minus 4x and du equals 3x squared minus 4.*
→ *Apply the log base a derivative formula with a equals 2.*
→ *Write the result as 3x squared minus 4 all over the quantity x cubed minus 4x times ln 2.*

**Answer:** $f'(x) = \dfrac{3x^2 - 4}{(x^3 - 4x)\ln 2}$

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**6.** Find the derivative of the function.

$y = \ln\left(\dfrac{x}{x+1}\right)$

→ *Expand using the quotient rule for logarithms to get ln x minus ln of x plus 1.*
→ *Differentiate each term to get 1 over x minus 1 over x plus 1.*
→ *Combine over a common denominator to get x plus 1 minus x over x times x plus 1.*
→ *Simplify the numerator to 1, giving 1 over x times x plus 1.*

**Answer:** $\dfrac{dy}{dx} = \dfrac{1}{x(x+1)}$

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**7.** Find the derivative of the function.

$f(x) = \ln(\cos x)$

→ *Identify u equals cosine x and du equals negative sine x.*
→ *Apply the natural log derivative formula du over u.*
→ *Get negative sine x over cosine x.*
→ *Recognize this as negative tangent x.*

**Answer:** $f'(x) = -\tan x$

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**8.** Find the derivative of the function.

$y = \log_5(4x^2 + 7)$

→ *Identify u equals 4x squared plus 7 and du equals 8x.*
→ *Apply the log base 5 derivative formula.*
→ *Write the simplified answer as 8x over the quantity 4x squared plus 7 times ln 5.*

**Answer:** $\dfrac{dy}{dx} = \dfrac{8x}{(4x^2 + 7)\ln 5}$

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**9.** Find the derivative of the function.

$f(x) = \ln(x^2 \cdot e^x)$

→ *Expand the logarithm using the product rule to get 2 ln x plus ln e to the x.*
→ *Simplify ln e to the x as just x.*
→ *Differentiate 2 ln x plus x to get 2 over x plus 1.*

**Answer:** $f'(x) = \dfrac{2}{x} + 1$

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**10.** Find the derivative of the function.

$y = \log_7(\tan x)$

→ *Identify u equals tangent x and du equals secant squared x.*
→ *Apply the log base 7 derivative formula.*
→ *Write the simplified result as secant squared x over the quantity tangent x times ln 7.*

**Answer:** $\frac{dy}{dx} = \frac{\sec^2 x}{\tan x \cdot \ln 7}$