Mean Value Theorem

Calculus Worksheet · Grade 11-12

Name: ______________________________ Date: _______________ Score: / 10

Learning Objectives

- Verify the hypotheses of the Mean Value Theorem on a closed interval
- Apply the Mean Value Theorem formula to locate the value c
- Interpret c as the point where the tangent line is parallel to the secant line

For each function and interval, verify the Mean Value Theorem applies, then find all values of c that satisfy the conclusion.

1. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^2 on [0,2].

$f(x) = x^2,\ [0, 2]$

Answer: ______________________________

2. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^2+2x on [-1,3].

$f(x) = x^2 + 2x,\ [-1, 3]$

Answer: ______________________________

3. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^3 on [0,3].

$f(x) = x^3,\ [0, 3]$

Answer: ______________________________

4. Find the value of c guaranteed by the Mean Value Theorem for f(x)=sqrt(x) on [1,9].

$f(x) = \sqrt{x},\ [1, 9]$

Answer: ______________________________

5. Find the value of c guaranteed by the Mean Value Theorem for f(x)=1/x on [1,4].

$f(x) = \frac{1}{x},\ [1, 4]$

Answer: ______________________________

6. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^3-x on [0,2].

$f(x) = x^3 - x,\ [0, 2]$

Answer: ______________________________

7. Find the value of c guaranteed by the Mean Value Theorem for f(x)=ln(x) on [1,e].

$f(x) = \ln(x),\ [1, e]$

Answer: ______________________________

8. Find the value of c guaranteed by the Mean Value Theorem for f(x)=2x^2-3x+1 on [1,4].

$f(x) = 2x^2 - 3x + 1, \ [1, 4]$

Answer: ____________________________________

9. Find the value of c guaranteed by the Mean Value Theorem for f(x)=sin(x) on [0,pi].

$f(x) = \sin(x), \ [0, \pi]$

Answer: ____________________________________

10. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^2-4x+3 on [1,5].

$f(x) = x^2 - 4x + 3, \ [1, 5]$

Answer: ____________________________________

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ANSWER KEY & SOLUTIONS

Remind students that the MVT requires continuity on [a,b] and differentiability on (a,b) before solving for c.

Solutions

1. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^2 on [0,2].

$f(x) = x^2,\ [0, 2]$

- → *Confirm f is continuous and differentiable everywhere, so MVT applies on the interval.*
- → *Compute f(b)=4 and f(a)=0, then form the average rate (4-0)/(2-0)=2.*
- → *Differentiate to get f prime of x equals 2x and set 2c equal to 2.*
- → *Solve to obtain c equals 1, which lies in the open interval (0,2).*

Answer: $c = 1$

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2. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^2+2x on [-1,3].

$f(x) = x^2 + 2x,\ [-1, 3]$

- → *The polynomial is continuous and differentiable on the interval, so MVT applies.*
- → *Evaluate f(3)=15 and f(-1)=-1, giving an average rate of (15-(-1))/(3-(-1))=4.*
- → *Differentiate to get f prime of x equals 2x+2 and set 2c+2 equal to 4.*
- → *Solve to obtain c equals 1, which lies in (-1,3).*

Answer: $c = 1$

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3. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^3 on [0,3].

$f(x) = x^3,\ [0, 3]$

- → *The cubic is continuous and differentiable on all real numbers.*
- → *Compute f(3)=27 and f(0)=0, giving an average rate of 27/3=9.*
- → *Differentiate to get f prime of x equals 3x^2 and set 3c^2 equal to 9.*
- → *Solve c^2=3 and take the positive root since c must lie in (0,3), giving c equals the square root of 3.*

Answer: $c = \sqrt{3}$

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4. Find the value of c guaranteed by the Mean Value Theorem for f(x)=sqrt(x) on [1,9].

$f(x) = \sqrt{x},\ [1, 9]$

- → *The function is continuous on [1,9] and differentiable on (1,9).*
- → *Compute f(9)=3 and f(1)=1, giving an average rate of (3-1)/(9-1)=1/4.*
- → *Differentiate to get f prime of x equals 1/(2 square root of x) and set it equal to 1/4.*
- → *Solve 2 square root of c equals 4, so square root of c equals 2 and c equals 4.*

Answer: $c = 4$

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5. Find the value of c guaranteed by the Mean Value Theorem for f(x)=1/x on [1,4].

$f(x) = \frac{1}{x},\ [1, 4]$

→ *The function is continuous and differentiable on the interval since x is never zero there.*
→ *Compute f(4)=1/4 and f(1)=1, giving an average rate of (1/4-1)/(4-1)=-1/4.*
→ *Differentiate to get f prime of x equals -1/x^2 and set -1/c^2 equal to -1/4.*
→ *Solve c^2=4 and take the positive root, giving c equals 2.*

Answer: $c = 2$

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6. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^3-x on [0,2].

$f(x) = x^3 - x,\ [0, 2]$

→ *The polynomial is continuous and differentiable everywhere.*
→ *Compute f(2)=6 and f(0)=0, giving an average rate of 6/2=3.*
→ *Differentiate to get f prime of x equals 3x^2-1 and set 3c^2-1 equal to 3.*
→ *Solve c^2=4/3, so c equals 2 over the square root of 3, which rationalizes to 2 square root of 3 divided by 3.*

Answer: $c = \frac{2\sqrt{3}}{3}$

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7. Find the value of c guaranteed by the Mean Value Theorem for f(x)=ln(x) on [1,e].

$f(x) = \ln(x),\ [1, e]$

→ *The natural log is continuous and differentiable on positive reals.*
→ *Compute f(e)=1 and f(1)=0, giving an average rate of (1-0)/(e-1)=1/(e-1).*
→ *Differentiate to get f prime of x equals 1/x and set 1/c equal to 1/(e-1).*
→ *Solve to obtain c equals e minus 1, which lies in (1,e).*

Answer: $c = e - 1$

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8. Find the value of c guaranteed by the Mean Value Theorem for f(x)=2x^2-3x+1 on [1,4].

$f(x) = 2x^2 - 3x + 1,\ [1, 4]$

→ *The polynomial is continuous and differentiable everywhere.*
→ *Compute f(4)=21 and f(1)=0, giving an average rate of (21-0)/(4-1)=7.*
→ *Differentiate to get f prime of x equals 4x-3 and set 4c-3 equal to 7.*
→ *Solve to obtain c equals 10/4 which simplifies to 5/2.*

Answer: $c = \frac{5}{2}$

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9. Find the value of c guaranteed by the Mean Value Theorem for f(x)=sin(x) on [0,pi].

$f(x) = \sin(x),\ [0, \pi]$

→ *Sine is continuous and differentiable on all real numbers.*
→ *Compute f(pi)=0 and f(0)=0, giving an average rate of (0-0)/(pi-0)=0.*
→ *Differentiate to get f prime of x equals cos(x) and set cos(c) equal to 0.*
→ *Solve for c in (0,pi) to obtain c equals pi over 2.*

Answer: $c = \frac{\pi}{2}$

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10. Find the value of c guaranteed by the Mean Value Theorem for f(x)=x^2-4x+3 on [1,5].

$f(x) = x^2 - 4x + 3,\ [1,5]$

→ *The polynomial is continuous and differentiable everywhere.*
→ *Compute f(5)=8 and f(1)=0, giving an average rate of (8-0)/(5-1)=2.*
→ *Differentiate to get f prime of x equals 2x-4 and set 2c-4 equal to 2.*
→ *Solve to obtain c equals 3, which lies in (1,5).*

Answer: $c = 3$