

Optimization & Absolute Extrema

Numberbender | WORKSHEET



Name: _____ Date: _____ Score: _____

Find all absolute and relative maximums and minimums. Show all work.

CALCULUS I Worksheet #67

1.	Find all absolute and relative maximums and minimums for $y = 2x^3 - 3x^2 - 12x$ on $[-2,3]$.																																				
2.	Find all critical points, inflection points, and sketch $y=x^4-4x^3$.																																				
3.	Find all intervals where $y = x^4 - 4x^3 + 4x^2$ is increasing and decreasing.																																				
4.	$f(x) = x^4 - 4x^2$ has: (A) 1 max, 2 min (B) 1 min, 2 max (C) 1 min, 1 max (D) 2 max, 0 min (E) 2 min, 0 max																																				
5.	The number of inflection points of the curve in problem 4 is: (A) 0 (B) 1 (C) 2 (D) 3 (E) 4																																				
6.	The total number of relative maximum and minimum points of the function whose derivative is $f'(x) = x^2(x+1)^3(x-4)^3$ is (A) 0 (B) 1 (C) 2 (D) 3 (E) 4																																				
7.	The sum of one number and twice another is 24. Find the two numbers so that their product is a maximum.																																				
8.	A square piece of tin has 12 inches on a side. An open box is formed by cutting out equal square pieces at the corners and bending upward the projecting portions which remain. Find the maximum volume that can be obtained.																																				
9.	Find all absolute and relative maximums and minimums for $y = x^3 - 3x + 1$ on $[-2,3]$.																																				
10.	Find the volume of the solid formed if the area bounded by $y = 4x - x^2$ and the x-axis is revolved around the y-axis.																																				
11.	f is continuous on $[0, 3]$ and satisfies the following: <table style="margin: 10px auto; border-collapse: collapse;"> <tbody> <tr> <td style="border: 1px solid black; padding: 2px;">x</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">3</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">f</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">-2</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">f'</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">DNE</td> <td style="border: 1px solid black; padding: 2px;">-3</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">f''</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">-1</td> <td style="border: 1px solid black; padding: 2px;">DNE</td> <td style="border: 1px solid black; padding: 2px;">0</td> </tr> </tbody> </table> <table style="margin: 10px auto; border-collapse: collapse;"> <thead> <tr> <th style="border: 1px solid black; padding: 2px;">x</th> <th style="border: 1px solid black; padding: 2px;">$0 < x < 1$</th> <th style="border: 1px solid black; padding: 2px;">$1 < x < 2$</th> <th style="border: 1px solid black; padding: 2px;">$2 < x < 3$</th> </tr> </thead> <tbody> <tr> <td style="border: 1px solid black; padding: 2px;">f</td> <td style="border: 1px solid black; padding: 2px;">+</td> <td style="border: 1px solid black; padding: 2px;">+</td> <td style="border: 1px solid black; padding: 2px;">-</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">f'</td> <td style="border: 1px solid black; padding: 2px;">+</td> <td style="border: 1px solid black; padding: 2px;">-</td> <td style="border: 1px solid black; padding: 2px;">-</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">f''</td> <td style="border: 1px solid black; padding: 2px;">-</td> <td style="border: 1px solid black; padding: 2px;">-</td> <td style="border: 1px solid black; padding: 2px;">-</td> </tr> </tbody> </table> <p>Find: (a) The absolute extrema of f and where they occur (b) Any inflection points (c) Sketch a possible graph of f.</p>	x	0	1	2	3	f	0	2	0	-2	f'	3	0	DNE	-3	f''	0	-1	DNE	0	x	$0 < x < 1$	$1 < x < 2$	$2 < x < 3$	f	+	+	-	f'	+	-	-	f''	-	-	-
x	0	1	2	3																																	
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f''	-	-	-																																		

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Numberbender | ANSWER KEY



Answer key — for instructor use only.

Answers:

1.	relative and absolute maximum $(-1,7)$, & relative and absolute minimum $(2,-20)$
2.	relative minimum $(3,-27)$; terrace point $(0,0)$; inflection point $(0,0)$, $(2,-16)$. There is also a graph.
3.	increasing: $(0, 1)$ and $(2, \infty)$; decreasing: $(-\infty, 2)$ and $(1, 2)$
4.	A
5.	C
6.	C
7.	12, 6 (product 72)
8.	128 in.^3
9.	absolute minimum $(-2,-1)$, relative and absolute minimum $(1,-1)$ absolute maximum $(3,19)$, relative maximum $(-1,3)$
10.	$\frac{128\pi}{3}$
11.	a) absolute max @ $(1, 2)$; absolute min @ $(2, -2)$