Name: _____ Date: _____ Score: _____

Solve each optimization problem. For graphing problems, find critical points, intervals of increase/decrease, and sketch the graph.

Calculus 1 Worksheet #66
Optimization and Review of Graphing

1*. The difference of two numbers is 50. Find the two numbers so that their product is as small as possible.

2*. Find two numbers whose sum is 36 and whose product is as large as possible.

3*. A farmer has 100 feet of fence and wishes to enclose a rectangular plot of land. The land borders a river so no fence is required on that side. What should the dimensions of the rectangle be in order that it include the largest possible area?

4. The slope of the line tangent to the curve $2x^3 - x^2y^2 + 4y^3 = 16$ at the point (2,1) is
A) –7 B) –5 C) –1 D) 5 E) 7

Find **and label** the intervals where the curve is increasing and decreasing, all critical points, and sketch the curve on graph paper. Make sure to show a chart.

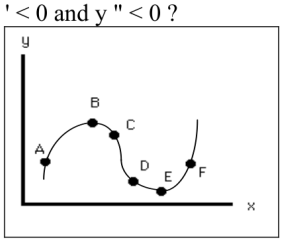
5. $f(x) = 3x^2 - 3x + 2$ 6. $f(x) = x^3 - x^2 - x$ 7. $f(x) = \dfrac{x-2}{x+2}$

8. Find a and b so that $f(x) = x^3 + ax^2 + b$ will have a critical point at (2,3).

9. Give the total number of max. and min. points of the function whose derivative is given by $f'(x) = x(x-3)^2(x+1)^4$.

10. Find all critical points and points of inflection, and sketch the curve.
(A) $f(x) = x^4 + 4x^3 + 5$ (B) $f(x) = x^4 - 2x^2$

11. Which point on the graph is $y' < 0$ and $y'' < 0$?



12. If $y = x^2 - 4x + 3$ where x can only be the numbers such that $0 \le x \le 5$, then what is the largest value that y will be?

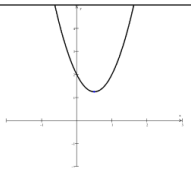
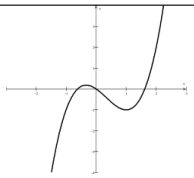
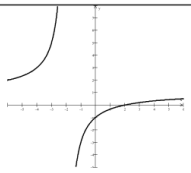
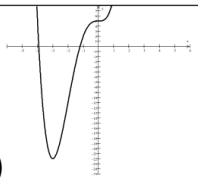
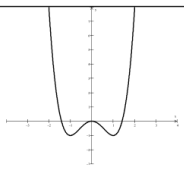
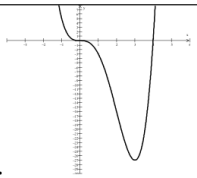
13. Find all absolute and relative maximums and minimums for $y = 2x^3 - 3x^2 - 12x$ on $[-2,3]$

14. Find all critical points, inflection points, and sketch $y = x^4 - 4x^3$.

15. $f(x) = x^4 - 4x^2$ has: (A) 1 max, 2 min (B) 1 min, 2 max (C) 1 min, 1 max
(D) 2 max, 0 min (E) 2 min, 0 max

Answer key — for instructor use only.

Answers:

1. x = 25; y = -25
2. x = 18; y = 18
3. w = 25 ft; l = 50 ft
4. **B**
5. inc: $\left(\frac{1}{2},\infty\right)$ dec: $\left(-\infty,\frac{1}{2}\right)$ min: $\left(\frac{1}{2},\frac{5}{4}\right)$ 
6. inc: $\left(-\infty,\frac{-1}{3}\right),(1,\infty)$ dec: $\left(\frac{-1}{3},1\right)$ max: $\left(\frac{-1}{3},\frac{5}{27}\right)$ min: $(1,-1)$ 
7. inc: $(-\infty,-2)(-2,\infty)$, NONE 
8. a = –3, b=7
9. one (min at x = 0)
10a. Rel. min. $(-3,\ -22)$ Saddle pt. $(0,5)$ Inf. pts. $(0,5),(-2,\ -11)$ 
10b. Rel. min. $(-1,\ -1),(1,-1)$ Rel. max. (0,0) Inf. pts. $\left(\pm\sqrt{\frac{1}{3}},\frac{-5}{9}\right)$ 
11. **C**
12. **8**
13. rel and abs max. $(-1,7)$ rel. and abs. min. $(2,-20)$
14. rel. min. $(3,-27)$ saddle pt. $(0,0)$ Inf. pt $(0,0),(2,-16)$. 
15. **A**