Speeding Up and Slowing Down: Particle Motion Analysis

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Determine intervals where a particle is speeding up or slowing down using velocity and acceleration sign charts
- Find critical values of velocity and acceleration functions by factoring
- Apply the rule that matching signs indicate speeding up and opposite signs indicate slowing down

For each problem, use a sign chart of the velocity and acceleration to determine when the particle is speeding up or slowing down.

1. Given the velocity function below, find the times when the velocity equals zero.

$v(t) = t^2 - 6t + 8$

Answer: _____

2. Find the acceleration function from the velocity function below.

$v(t) = t^2 - 6t + 8$

Answer: _____

3. Determine the time at which the acceleration equals zero.

$a(t) = 2t - 6$

Answer: _____

4. Determine the sign of the velocity on the interval shown below.

$v(t) = t^2 - 6t + 8, \quad t \in (-\infty, 2)$

Answer: _____

5. Determine the sign of the velocity on the interval shown below.

$v(t) = t^2 - 6t + 8, \quad t \in (2, 4)$

Answer: _____

6. Determine whether the particle is speeding up or slowing down on the interval below.

$t \in (2, 3)$

Answer: _____

7. Determine whether the particle is speeding up or slowing down on the interval below.

$t \in (3, 4)$

Answer: _____

Scan to watch

8. Determine whether the particle is speeding up or slowing down on the interval below.

$t \in (4, \infty)$

Answer: _____

9. Determine whether the particle is speeding up or slowing down on the interval below.

$t \in (-\infty, 2)$

Answer: _____

10. State the complete interval(s) on which the particle is speeding up.

$v(t) = t^2 - 6t + 8$

Answer: _____

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ANSWER KEY & SOLUTIONS

Emphasize that speeding up occurs when velocity and acceleration have the same sign, while slowing down occurs when they have opposite signs.

Solutions

1. Given the velocity function below, find the times when the velocity equals zero.

$v(t) = t^2 - 6t + 8$

→ *Set the velocity function equal to zero*
→ *Factor the quadratic expression into two binomials*
→ *Apply the zero product property to find the two values of t*

Answer: $t = 2,\ t = 4$

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2. Find the acceleration function from the velocity function below.

$v(t) = t^2 - 6t + 8$

→ *Take the derivative of the velocity function with respect to t*
→ *Apply the power rule to each term*
→ *Simplify to obtain the acceleration function*

Answer: $a(t) = 2t - 6$

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3. Determine the time at which the acceleration equals zero.

$a(t) = 2t - 6$

→ *Set the acceleration function equal to zero*
→ *Add 6 to both sides of the equation*
→ *Divide both sides by 2 to solve for t*

Answer: $t = 3$

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4. Determine the sign of the velocity on the interval shown below.

$v(t) = t^2 - 6t + 8,\quad t \in (-\infty, 2)$

→ *Choose a test value less than 2, such as t equals 0*
→ *Substitute the test value into the velocity function*
→ *Evaluate to confirm the result is positive*

Answer: Positive

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5. Determine the sign of the velocity on the interval shown below.

$v(t) = t^2 - 6t + 8,\quad t \in (2, 4)$

→ *Choose a test value between 2 and 4, such as t equals 3*
→ *Substitute the test value into the velocity function*
→ *Evaluate to confirm the result is negative*

Answer: Negative

Scan to watch

6. Determine whether the particle is speeding up or slowing down on the interval below.

$t \in (2, 3)$

→ *Identify that the velocity is negative on this interval*
→ *Identify that the acceleration is negative on this interval since t is less than 3*
→ *Compare the signs and conclude the motion based on the matching or opposite signs rule*

Answer: Slowing down

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7. Determine whether the particle is speeding up or slowing down on the interval below.

$t \in (3, 4)$

→ *Identify that the velocity is negative on this interval*
→ *Identify that the acceleration is positive on this interval since t is greater than 3*
→ *Compare the signs and conclude the motion based on the matching or opposite signs rule*

Answer: Speeding up

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8. Determine whether the particle is speeding up or slowing down on the interval below.

$t \in (4, \infty)$

→ *Identify that the velocity is positive on this interval using a test value such as 5*
→ *Identify that the acceleration is positive on this interval since t is greater than 3*
→ *Compare the signs and conclude the motion based on the matching or opposite signs rule*

Answer: Speeding up

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9. Determine whether the particle is speeding up or slowing down on the interval below.

$t \in (-\infty, 2)$

→ *Identify that the velocity is positive on this interval using a test value such as 0*
→ *Identify that the acceleration is negative on this interval since t is less than 3*
→ *Compare the signs and conclude the motion based on the matching or opposite signs rule*

Answer: Slowing down

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10. State the complete interval(s) on which the particle is speeding up.

$v(t) = t^2 - 6t + 8$

→ *List all subintervals created by the critical values 2, 3, and 4*
→ *Identify each subinterval where velocity and acceleration share the same sign*
→ *Combine those intervals to express the full speeding up region*

Answer: $(3, 4) \cup (4, \infty)$