Particle Motion: Velocity and Acceleration from Position Functions

Calculus Worksheet · Grade 11-12

Name: ______________________________ Date: _______________ Score: / 10

Learning Objectives

- Compute velocity as the first derivative of a position function
- Compute acceleration as the second derivative of a position function
- Determine when a particle is at rest by solving v(t) = 0

For each position function s(t), find the requested velocity, acceleration, or time(s) at rest, showing all derivative work.

1. Given the position function below (s in meters, t in seconds), find the velocity after 3 seconds.

$$s(t) = 2t^3 - 7t^2 + 4t + 1$$

Answer: _________________________________

2. Using the same position function, find the acceleration after 1 second.

$$s(t) = 2t^3 - 7t^2 + 4t + 1$$

Answer: _________________________________

3. Using the same position function, determine the time(s) when the particle is at rest.

$$s(t) = 2t^3 - 7t^2 + 4t + 1$$

Answer: _________________________________

4. Find the velocity function and evaluate it at t = 2 seconds for the position function below.

$$s(t) = t^3 - 6t^2 + 9t + 5$$

Answer: _________________________________

5. Find the acceleration after 4 seconds for the position function below.

$$s(t) = t^3 - 6t^2 + 9t + 5$$

Answer: _________________________________

6. Determine the time(s) when the particle described below is at rest.

$$s(t) = t^3 - 6t^2 + 9t + 5$$

Answer: _________________________________

7. Find the velocity after 5 seconds for the position function below.

$$s(t) = 4t^2 - 3t + 7$$

Answer: _________________________________

8. Find the acceleration function and its value at t = 2 seconds for the position function below.

$$s(t) = 4t^2 - 3t + 7$$

Answer: ____________________________________

9. Determine the time when the particle below is at rest.

$$s(t) = t^2 - 10t + 12$$

Answer: ____________________________________

10. For the position function below, find both the velocity and acceleration at t = 1 second.

$$s(t) = t^4 - 4t^3 + 6t$$

Answer: ____________________________________

Students should recall the power rule and zero product property; remind them that 'at rest' means v(t) = 0, not s(t) = 0.

Solutions

1. Given the position function below (s in meters, t in seconds), find the velocity after 3 seconds.

$$s(t) = 2t^3 - 7t^2 + 4t + 1$$

→ *Differentiate s(t) using the power rule to obtain v(t) = 6t^2 - 14t + 4.*
→ *Substitute t = 3: v(3) = 6(9) - 14(3) + 4 = 54 - 42 + 4 = 16 m/s.*

Answer: $v(3) = 16\ \text{m/s}$

2. Using the same position function, find the acceleration after 1 second.

$$s(t) = 2t^3 - 7t^2 + 4t + 1$$

→ *Differentiate v(t) = 6t^2 - 14t + 4 to get a(t) = 12t - 14.*
→ *Substitute t = 1: a(1) = 12(1) - 14 = -2 m/s^2.*

Answer: $a(1) = -2\ \text{m/s}^2$

3. Using the same position function, determine the time(s) when the particle is at rest.

$$s(t) = 2t^3 - 7t^2 + 4t + 1$$

→ *Set v(t) = 0: 6t^2 - 14t + 4 = 0, which factors as (3t - 1)(t - 2) = 0 after dividing by 2.*
→ *Apply the zero product property: t = 1/3 second and t = 2 seconds.*

Answer: $t = \frac{1}{3}\ \text{s},\ t = 2\ \text{s}$

4. Find the velocity function and evaluate it at t = 2 seconds for the position function below.

$$s(t) = t^3 - 6t^2 + 9t + 5$$

→ *Differentiate to get v(t) = 3t^2 - 12t + 9.*
→ *Substitute t = 2: v(2) = 3(4) - 12(2) + 9 = 12 - 24 + 9 = -3 m/s.*

Answer: $v(2) = -3\ \text{m/s}$

5. Find the acceleration after 4 seconds for the position function below.

$$s(t) = t^3 - 6t^2 + 9t + 5$$

→ *Differentiate v(t) = 3t^2 - 12t + 9 to get a(t) = 6t - 12.*
→ *Substitute t = 4: a(4) = 6(4) - 12 = 12 m/s^2.*

Answer: $a(4) = 12\ \text{m/s}^2$

6. Determine the time(s) when the particle described below is at rest.

$$s(t) = t^3 - 6t^2 + 9t + 5$$

→ *Set v(t) = 3t^2 - 12t + 9 = 0 and divide by 3 to get t^2 - 4t + 3 = 0.*
→ *Factor as (t - 1)(t - 3) = 0, giving t = 1 second and t = 3 seconds.*

Answer: $t = 1\ \text{s},\ t = 3\ \text{s}$

7. Find the velocity after 5 seconds for the position function below.

$s(t) = 4t^2 - 3t + 7$

→ *Differentiate to get v(t) = 8t - 3.*
→ *Substitute t = 5: v(5) = 8(5) - 3 = 37 m/s.*

Answer: $v(5) = 37\ \text{m/s}$

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8. Find the acceleration function and its value at t = 2 seconds for the position function below.

$s(t) = 4t^2 - 3t + 7$

→ *Differentiate v(t) = 8t - 3 to get a(t) = 8.*
→ *Since a(t) is constant, a(2) = 8 m/s^2.*

Answer: $a(2) = 8\ \text{m/s}^2$

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9. Determine the time when the particle below is at rest.

$s(t) = t^2 - 10t + 12$

→ *Differentiate to get v(t) = 2t - 10.*
→ *Set 2t - 10 = 0 and solve to obtain t = 5 seconds.*

Answer: $t = 5\ \text{s}$

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10. For the position function below, find both the velocity and acceleration at t = 1 second.

$s(t) = t^4 - 4t^3 + 6t$

→ *Differentiate s(t) to get v(t) = 4t^3 - 12t^2 + 6, then substitute t = 1 to find v(1) = 4 - 12 + 6 = -2 m/s.*
→ *Differentiate again to get a(t) = 12t^2 - 24t, then substitute t = 1 to find a(1) = 12 - 24 = -12 m/s^2.*

Answer: $v(1) = -2\ \text{m/s},\ a(1) = -12\ \text{m/s}^2$