# Derivative Rules: Product & Quotient Rules with Given Values

Calculus Worksheet · Grade 11–12 / College Intro Calculus

Name: _____ Date: _____

**Learning Objectives**

- Apply the product rule to find derivatives of products of two functions at a given point
- Apply the quotient rule to find derivatives of quotients of two functions at a given point
- Combine derivative rules (constant multiple, sum/difference, product) to differentiate composite expressions

## Problems

**1.** Let U and V be differentiable functions of x with U(3) = 5, U'(3) = -2, V(3) = 4, and V'(3) = 3. Find the derivative of the product UV evaluated at x = 3.

**2.** Using the same values as Problem 1 — U(3) = 5, U'(3) = -2, V(3) = 4, V'(3) = 3 — find the derivative of U divided by V, evaluated at x = 3.

**3.** Using the values U(3) = 5, U'(3) = -2, V(3) = 4, V'(3) = 3, find the derivative of V divided by U, evaluated at x = 3.

**4.** Suppose U and V are differentiable at x = 2 with U(2) = 3, U'(2) = -4, V(2) = 1, and V'(2) = 2. Find the derivative of 3U minus 2V plus UV, evaluated at x = 2.

**5.** Let U and V be differentiable at x = 1 with U(1) = -2, U'(1) = 5, V(1) = 3, and V'(1) = -1. Find the derivative of 4U plus UV, evaluated at x = 1.

**6.** Let U and V be differentiable at x = 0 with U(0) = 6, U'(0) = 2, V(0) = -3, and V'(0) = 4. Find the derivative of U times V, then find the derivative of U divided by V, both evaluated at x = 0. Report both answers.

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**7.** Suppose U and V are differentiable at x = 5 with U(5) = -1, U'(5) = 3, V(5) = 2, and V'(5) = -5. Find the derivative of 2UV minus V divided by U, evaluated at x = 5.

**8.** Let U and V be differentiable at x = 4 with U(4) = 7, U'(4) = -3, V(4) = 5, and V'(4) = 1. Find the derivative of U squared times V, evaluated at x = 4. Hint: treat U squared as U times U and use the product rule twice.

**9.** Suppose U and V are differentiable at x = 2 with U(2) = 3, U'(2) = -4, V(2) = 1, and V'(2) = 2. Find the derivative of the expression 5U divided by the quantity U minus V, evaluated at x = 2.

**10.** Let U and V be differentiable at x = 3 with U(3) = 2, U'(3) = 6, V(3) = -4, and V'(3) = 3. Find the derivative of the expression U times V divided by U plus V, evaluated at x = 3. This requires both the product rule and the quotient rule.

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# Derivative Rules: Product & Quotient Rules with Given Values — Answer Key

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## Answer Key

**1. Answer: 7**
- Product Rule: d/dx[UV] = U·V' + U'·V
- Substitute values at x = 3: (5)(3) + (-2)(4)
- = 15 + (-8)
- = 7

**2. Answer: 23/16**
- Quotient Rule: d/dx[U/V] = (V·U' – U·V') / V²
- Substitute at x = 3: (4·(–2) – 5·3) / (4²)
- = (–8 – 15) / 16
- = –23 / 16

**3. Answer: 23/25**
- Quotient Rule: d/dx[V/U] = (U·V' – V·U') / U²
- Substitute at x = 3: (5·3 – 4·(–2)) / (5²)
- = (15 + 8) / 25
- = 23/25

**4. Answer: -14**
- Differentiate term by term: 3U' – 2V' + (U'V + UV')
- Substitute at x = 2: 3(–4) – 2(2) + ((–4)(1) + (3)(2))
- = –12 – 4 + (–4 + 6)
- = –12 – 4 + 2 = –14

**5. Answer: 19**
- Differentiate: 4U' + (U'V + UV')
- Substitute at x = 1: 4(5) + (5·3 + (–2)·(–1))
- = 20 + (15 + 2)
- = 20 + 17 = 37
- Wait — recheck: 4(5) = 20; U'V = 5·3 = 15; UV' = (–2)(–1) = 2; Total = 20 + 15 + 2 = 37
- Answer: 37

**6. Answer: d/dx[UV] = 18; d/dx[U/V] = -20/9**
- Product Rule: UV' + U'V = (6)(4) + (2)(–3) = 24 – 6 = 18
- Quotient Rule: (V·U' – U·V') / V² = ((–3)(2) – (6)(4)) / (–3)²
- = (–6 – 24) / 9 = –30/9 = –10/3
- d/dx[UV] = 18 and d/dx[U/V] = –10/3

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**7. Answer: -17**

- Differentiate 2UV: 2(U'V + UV') = 2((3)(2) + (–1)(–5)) = 2(6 + 5) = 22
- Differentiate V/U using quotient rule: (U·V' – V·U') / U²
- = ((–1)(–5) – (2)(3)) / (–1)² = (5 – 6) / 1 = –1
- Combine: 22 – (–1) = 23
- Wait — re-evaluate: d/dx[2UV] = 22, d/dx[V/U] = –1, so 22 – (–1) = 23
- Answer: 23

**8. Answer: -161**

- Write U²V = (U·U)·V, then use product rule on U·U first: d/dx[U²] = 2U·U'
- Now apply product rule to U²·V: d/dx[U²V] = (d/dx[U²])·V + U²·V'
- = 2U·U'·V + U²·V'
- Substitute at x = 4: 2(7)(–3)(5) + (7²)(1)
- = 2(7)(–3)(5) + 49 = –210 + 49 = –161

**9. Answer: -26/4 = -13/2**

- Let numerator = 5U, denominator = U – V
- d/dx[5U] = 5U'; d/dx[U – V] = U' – V'
- Quotient Rule: [(U–V)·5U' – 5U·(U'–V')] / (U–V)²
- At x = 2: U–V = 3–1 = 2; (U–V)² = 4
- Numerator: (2)(5)(–4) – (5)(3)((–4)–2) = –40 – (15)(–6) = –40 + 90 = 50
- Wait — recompute: (U–V)·5U' = 2·5·(–4) = –40; 5U·(U'–V') = 5·3·(–4–2) = 15·(–6) = –90
- Numerator = –40 – (–90) = –40 + 90 = 50; Result = 50/4 = 25/2
- Answer: 25/2

**10. Answer: 66/4 = 33/2**

- Let numerator f = UV and denominator g = U + V
- f' = U'V + UV' = (6)(–4) + (2)(3) = –24 + 6 = –18
- g' = U' + V' = 6 + 3 = 9
- At x = 3: f = UV = (2)(–4) = –8; g = U + V = 2 + (–4) = –2
- Quotient Rule: (g·f' – f·g') / g²
- = ((–2)(–18) – (–8)(9)) / (–2)²
- = (36 + 72) / 4
- = 108 / 4 = 27

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