

# Finding the Derivative Using the Product Rule

Calculus Worksheet · Grade 11–12 / College

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Learning Objectives

- Identify when the product rule is needed for differentiation
- Apply the product rule formula  $d/dx[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$  correctly
- Simplify derivatives obtained through the product rule

## Problems

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1. Find the derivative of the function using the product rule.

$$y = x^2 \cdot x^3$$

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2. Use the product rule to differentiate the function.

$$y = 3x \cdot (x + 4)$$

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3. Find the derivative of the function using the product rule.

$$f(x) = x^4 \cdot (2x - 1)$$

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4. Differentiate the function using the product rule.

$$y = (x^2 + 3)(x^2 - 5)$$

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5. Use the product rule to find the derivative of the function.

$$f(x) = (5x^3 + 2x)(x^2 - 4)$$

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6. Find the derivative of the function involving a square root using the product rule.

$$y = x^3 \cdot \sqrt{x}$$

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7. Use the product rule to differentiate the function containing a trigonometric factor.

$$f(x) = x^2 \sin(x)$$

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8. Find the derivative of the function using the product rule.

$$y = (3x^2 - x + 2)(4x^3 + 7)$$

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9. Use the product rule to find the derivative, then evaluate it at the given value of  $x$ .

$$f(x) = (x^2 + 1)(3x - 2), \quad x = 2$$

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10. Apply the product rule twice (extended product rule) to differentiate the function involving three factors.

$$y = x^2(x + 1)(2x - 3)$$

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# Finding the Derivative Using the Product Rule — Answer Key

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## Answer Key

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### 1. Answer: $y' = 5x^4$

- Let  $f(x) = x^2$  and  $g(x) = x^3$
  - Find  $f'(x) = 2x$  and  $g'(x) = 3x^2$
  - Apply product rule:  $y' = f'(x) \cdot g(x) + f(x) \cdot g'(x) = 2x \cdot x^3 + x^2 \cdot 3x^2$
  - Simplify:  $y' = 2x^4 + 3x^4 = 5x^4$
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### 2. Answer: $y' = 6x + 12$

- Let  $f(x) = 3x$  and  $g(x) = x + 4$
  - Find  $f'(x) = 3$  and  $g'(x) = 1$
  - Apply product rule:  $y' = 3 \cdot (x + 4) + 3x \cdot 1$
  - Simplify:  $y' = 3x + 12 + 3x = 6x + 12$
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### 3. Answer: $f'(x) = 10x^4 - 4x^3$

- Let  $u = x^4$  and  $v = 2x - 1$
  - Find  $u' = 4x^3$  and  $v' = 2$
  - Apply product rule:  $f'(x) = 4x^3 \cdot (2x - 1) + x^4 \cdot 2$
  - Expand:  $f'(x) = 8x^4 - 4x^3 + 2x^4$
  - Simplify:  $f'(x) = 10x^4 - 4x^3$
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### 4. Answer: $y' = 4x^3 - 4x$

- Let  $f(x) = x^2 + 3$  and  $g(x) = x^2 - 5$
  - Find  $f'(x) = 2x$  and  $g'(x) = 2x$
  - Apply product rule:  $y' = 2x \cdot (x^2 - 5) + (x^2 + 3) \cdot 2x$
  - Expand:  $y' = 2x^3 - 10x + 2x^3 + 6x$
  - Simplify:  $y' = 4x^3 - 4x$
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### 5. Answer: $f'(x) = 25x^4 - 54x^2 - 8$

- Let  $u = 5x^3 + 2x$  and  $v = x^2 - 4$
  - Find  $u' = 15x^2 + 2$  and  $v' = 2x$
  - Apply product rule:  $f'(x) = (15x^2 + 2)(x^2 - 4) + (5x^3 + 2x)(2x)$
  - Expand first term:  $15x^4 - 60x^2 + 2x^2 - 8 = 15x^4 - 58x^2 - 8$
  - Expand second term:  $10x^4 + 4x^2$
  - Combine:  $f'(x) = 25x^4 - 54x^2 - 8$
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### 6. Answer: $y' = (7/2)x^{(5/2)}$

- Rewrite:  $y = x^3 \cdot x^{(1/2)}$
- Let  $f(x) = x^3$  and  $g(x) = x^{(1/2)}$

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- Find  $f'(x) = 3x^2$  and  $g'(x) = (1/2)x^{-1/2}$
- Apply product rule:  $y' = 3x^2 \cdot x^{1/2} + x^3 \cdot (1/2)x^{-1/2}$
- Simplify each term:  $3x^{5/2} + (1/2)x^{5/2}$
- Combine:  $y' = (7/2)x^{5/2}$

**7. Answer:  $f'(x) = 2x \sin(x) + x^2 \cos(x)$**

- Let  $u = x^2$  and  $v = \sin(x)$
- Find  $u' = 2x$  and  $v' = \cos(x)$
- Apply product rule:  $f'(x) = 2x \cdot \sin(x) + x^2 \cdot \cos(x)$
- The expression is fully simplified:  $f'(x) = 2x \sin(x) + x^2 \cos(x)$

**8. Answer:  $y' = 60x^4 - 16x^3 + 24x^2 - 7 + 42x$**

- Let  $f(x) = 3x^2 - x + 2$  and  $g(x) = 4x^3 + 7$
- Find  $f'(x) = 6x - 1$  and  $g'(x) = 12x^2$
- Apply product rule:  $y' = (6x - 1)(4x^3 + 7) + (3x^2 - x + 2)(12x^2)$
- Expand first term:  $24x^4 + 42x - 4x^3 - 7$
- Expand second term:  $36x^4 - 12x^3 + 24x^2$
- Combine like terms:  $y' = 60x^4 - 16x^3 + 24x^2 + 42x - 7$

**9. Answer:  $f'(2) = 35$**

- Let  $u = x^2 + 1$  and  $v = 3x - 2$
- Find  $u' = 2x$  and  $v' = 3$
- Apply product rule:  $f'(x) = 2x(3x - 2) + (x^2 + 1)(3)$
- Expand:  $f'(x) = 6x^2 - 4x + 3x^2 + 3 = 9x^2 - 4x + 3$
- Evaluate at  $x = 2$ :  $f'(2) = 9(4) - 4(2) + 3 = 36 - 8 + 3 = 31$
- Correction check — re-expand:  $6x^2 - 4x + 3x^2 + 3 = 9x^2 - 4x + 3$ ;  $f'(2) = 36 - 8 + 3 = 31$
- Answer:  $f'(2) = 31$

**10. Answer:  $y' = 8x^3 - 3x^2 - 6x$**

- First, group the function as  $y = x^2 \cdot [(x + 1)(2x - 3)]$
- Find the derivative of the inner product  $h(x) = (x + 1)(2x - 3)$
- $h'(x) = 1 \cdot (2x - 3) + (x + 1) \cdot 2 = 2x - 3 + 2x + 2 = 4x - 1$
- Now apply the product rule to  $y = x^2 \cdot h(x)$ :  $y' = 2x \cdot h(x) + x^2 \cdot h'(x)$
- Substitute  $h(x) = (x + 1)(2x - 3) = 2x^2 - x - 3$  and  $h'(x) = 4x - 1$
- $y' = 2x(2x^2 - x - 3) + x^2(4x - 1)$
- Expand:  $y' = 4x^3 - 2x^2 - 6x + 4x^3 - x^2$
- Combine:  $y' = 8x^3 - 3x^2 - 6x$

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