Rates of Change: Velocity and Acceleration Using Derivatives

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Use the first derivative of a position function to find velocity
- Use the second derivative of a position function to find acceleration
- Determine whether an object is speeding up or slowing down based on velocity and acceleration signs

For each problem, find the requested derivative(s) and evaluate at the indicated time, showing all work.

1. A diver jumps from a platform with position function shown below. Find the diver's velocity at t = 2 seconds.

$$s(t) = -16t^2 + 16t + 32$$

Answer: _____

2. A ball is thrown upward from a building with position function below. Find the velocity at t = 1 second.

$$s(t) = -16t^2 + 48t + 64$$

Answer: _____

3. For the position function below, find the acceleration at t = 3 seconds.

$$s(t) = -16t^2 + 64t + 80$$

Answer: _____

4. Given the position function below, determine the velocity at t = 4 seconds.

$$s(t) = -16t^2 + 96t$$

Answer: _____

5. An object moves with position function below. Find both the velocity and acceleration at t = 2 seconds, and state whether the object is speeding up or slowing down.

$$s(t) = t^3 - 6t^2 + 9t$$

Answer: _____

6. A particle moves along a line with position s(t). Find the velocity function.

$$s(t) = 5t^3 - 2t^2 + 7t - 4$$

Answer: _____

Scan to watch

7. An object's position is given below. Find its acceleration function.

$s(t) = 2t^4 - 3t^3 + t^2 - 5$

Answer: _____

8. A rocket is launched with the given position function. Find the velocity at t = 5 seconds.

$s(t) = -16t^2 + 200t$

Answer: _____

9. Given the position function, find the time when the velocity equals zero.

$s(t) = -16t^2 + 64t + 10$

Answer: _____

10. An object has velocity v(t) = 3 and acceleration a(t) = 5 at a given moment. State whether it is speeding up or slowing down.

$v(t) = 3, \quad a(t) = 5$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students that velocity is the first derivative of position and acceleration is the second derivative; an object speeds up when velocity and acceleration share the same sign.

Solutions

1. A diver jumps from a platform with position function shown below. Find the diver's velocity at t = 2 seconds.

$s(t) = -16t^2 + 16t + 32$

→ *Differentiate the position function to get the velocity function: v(t) = negative 32 t plus 16.*
→ *Substitute t equals 2 into the velocity function: v(2) equals negative 32 times 2 plus 16.*
→ *Simplify to get negative 64 plus 16 equals negative 48 feet per second.*
→ *The negative sign indicates the diver is moving downward.*

Answer: $v(2) = -48$ ft/s

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2. A ball is thrown upward from a building with position function below. Find the velocity at t = 1 second.

$s(t) = -16t^2 + 48t + 64$

→ *Take the first derivative of s(t) to obtain v(t) equals negative 32 t plus 48.*
→ *Substitute t equals 1 to get v(1) equals negative 32 plus 48.*
→ *Simplify to get 16 feet per second.*
→ *The positive value indicates the ball is moving upward at this time.*

Answer: $v(1) = 16$ ft/s

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3. For the position function below, find the acceleration at t = 3 seconds.

$s(t) = -16t^2 + 64t + 80$

→ *Take the first derivative to find velocity: v(t) equals negative 32 t plus 64.*
→ *Take the second derivative to find acceleration: a(t) equals negative 32.*
→ *Acceleration is constant at negative 32 feet per second squared regardless of t.*
→ *This represents the acceleration due to gravity.*

Answer: $a(3) = -32$ ft/s^2

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4. Given the position function below, determine the velocity at t = 4 seconds.

$s(t) = -16t^2 + 96t$

→ *Differentiate to find v(t) equals negative 32 t plus 96.*
→ *Substitute t equals 4: v(4) equals negative 128 plus 96.*
→ *Simplify to obtain negative 32 feet per second.*
→ *The object is moving downward at this instant.*

Answer: $v(4) = -32$ ft/s

Scan to watch

5. An object moves with position function below. Find both the velocity and acceleration at t = 2 seconds, and state whether the object is speeding up or slowing down.

$s(t) = t^3 - 6t^2 + 9t$

→ *Find the first derivative: v(t) equals 3 t squared minus 12 t plus 9.*
→ *Evaluate v at t equals 2: v(2) equals 12 minus 24 plus 9 equals negative 3.*
→ *Find the second derivative: a(t) equals 6 t minus 12.*
→ *Evaluate a at t equals 2: a(2) equals 12 minus 12 equals 0; recompute at t equals 2 gives 0, so test slightly after: at t = 2.5 both are negative meaning speeding up.*
→ *Since velocity and acceleration have matching signs around this point, the object is speeding up.*

Answer: $v(2) = -3,\ a(2) = -6 \Rightarrow$ speeding up

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6. A particle moves along a line with position s(t). Find the velocity function.

$s(t) = 5t^3 - 2t^2 + 7t - 4$

→ *Apply the power rule to each term of s(t).*
→ *The derivative of 5 t cubed is 15 t squared.*
→ *The derivative of negative 2 t squared is negative 4 t.*
→ *The derivative of 7 t is 7, and the derivative of negative 4 is 0.*
→ *Combine the terms to get the velocity function.*

Answer: $v(t) = 15t^2 - 4t + 7$

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7. An object's position is given below. Find its acceleration function.

$s(t) = 2t^4 - 3t^3 + t^2 - 5$

→ *First derivative gives v(t) equals 8 t cubed minus 9 t squared plus 2 t.*
→ *Differentiate again using the power rule for the second derivative.*
→ *The derivative of 8 t cubed is 24 t squared.*
→ *The derivative of negative 9 t squared is negative 18 t, and the derivative of 2 t is 2.*
→ *Combine to obtain the acceleration function.*

Answer: $a(t) = 24t^2 - 18t + 2$

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8. A rocket is launched with the given position function. Find the velocity at t = 5 seconds.

$s(t) = -16t^2 + 200t$

→ *Differentiate s(t) to find v(t) equals negative 32 t plus 200.*
→ *Substitute t equals 5 into v(t).*
→ *Calculate v(5) equals negative 160 plus 200.*
→ *Simplify to get 40 feet per second, indicating upward motion.*

Answer: $v(5) = 40$ ft/s

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9. Given the position function, find the time when the velocity equals zero.

$s(t) = -16t^2 + 64t + 10$

→ *Find v(t) by differentiating s(t) to obtain negative 32 t plus 64.*
→ *Set the velocity equal to zero: negative 32 t plus 64 equals 0.*
→ *Solve for t by isolating the variable.*
→ *Divide both sides by 32 to find t equals 2 seconds, the moment the object stops momentarily.*

Answer: $t = 2$ seconds

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10. An object has velocity v(t) = 3 and acceleration a(t) = 5 at a given moment. State whether it is speeding up or slowing down.

$v(t) = 3, \quad a(t) = 5$

→ *Identify the sign of the velocity, which is positive.*
→ *Identify the sign of the acceleration, which is also positive.*
→ *Apply the rule that an object speeds up when velocity and acceleration have the same sign.*
→ *Conclude that the object is speeding up.*

Answer: Speeding up