Rates of Change: Velocity and Acceleration

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Apply the first derivative to find velocity of a moving object
- Apply the second derivative to find acceleration of a moving object
- Determine whether an object is speeding up or slowing down using velocity and acceleration

For each problem, find the requested derivative or evaluate it at the given time, and interpret the result in terms of motion.

1. A diver jumps from a 32 ft platform. The position function is given below. Find the velocity function.

$$f(t) = -16t^2 + 16t + 32$$

Answer: _____

2. Using the velocity function from the diver problem, find the diver's velocity at t = 2 seconds.

$$v(t) = -32t + 16, \quad t = 2$$

Answer: _____

3. Find the acceleration function for the diver by taking the second derivative of the position function.

$$f(t) = -16t^2 + 16t + 32$$

Answer: _____

4. A ball is thrown upward with position function below. Find the velocity at t = 1 second.

$$s(t) = -16t^2 + 48t + 6$$

Answer: _____

5. For the position function below, find the velocity at t = 3.

$$s(t) = t^3 - 6t^2 + 9t$$

Answer: _____

6. Using the position function s(t) = t^3 - 6t^2 + 9t, find the acceleration at t = 3.

$$s(t) = t^3 - 6t^2 + 9t, \quad t = 3$$

Answer: _____

Scan to watch

7. At t = 2 seconds, an object has velocity v = 5 and acceleration a = 3. Determine whether the object is speeding up or slowing down.

$v(2) = 5, \quad a(2) = 3$

Answer: _____

8. At t = 4 seconds, an object has velocity v = -8 and acceleration a = -2. Determine whether the object is speeding up or slowing down.

$v(4) = -8, \quad a(4) = -2$

Answer: _____

9. At t = 1 second, an object has velocity v = 6 and acceleration a = -4. Determine whether the object is speeding up or slowing down.

$v(1) = 6, \quad a(1) = -4$

Answer: _____

10. A particle moves along a line with position function below. Find the velocity function v(t).

$s(t) = 2t^3 - 9t^2 + 12t - 4$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students that the first derivative gives velocity and the second derivative gives acceleration; an object speeds up when velocity and acceleration share the same sign.

Solutions

1. A diver jumps from a 32 ft platform. The position function is given below. Find the velocity function.

$f(t) = -16t^2 + 16t + 32$

→ *Take the first derivative of the position function with respect to t.*
→ *The derivative of negative sixteen t squared is negative thirty-two t.*
→ *The derivative of sixteen t is sixteen, and the derivative of the constant thirty-two is zero.*
→ *Combine the terms to get the velocity function.*

Answer: $v(t) = -32t + 16$

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2. Using the velocity function from the diver problem, find the diver's velocity at t = 2 seconds.

$v(t) = -32t + 16, \quad t = 2$

→ *Substitute t equals two into the velocity function.*
→ *Multiply negative thirty-two by two to get negative sixty-four.*
→ *Add sixteen to negative sixty-four to get negative forty-eight.*
→ *The negative sign indicates the diver is moving downward.*

Answer: $v(2) = -48$ ft/s

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3. Find the acceleration function for the diver by taking the second derivative of the position function.

$f(t) = -16t^2 + 16t + 32$

→ *First find the first derivative which is the velocity function.*
→ *Take the derivative of the velocity function with respect to t.*
→ *The derivative of negative thirty-two t is negative thirty-two.*
→ *The derivative of the constant sixteen is zero, giving a constant acceleration.*

Answer: $a(t) = -32$ ft/s^2

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4. A ball is thrown upward with position function below. Find the velocity at t = 1 second.

$s(t) = -16t^2 + 48t + 6$

→ *Take the first derivative to find the velocity function.*
→ *The velocity function is negative thirty-two t plus forty-eight.*
→ *Substitute t equals one into the velocity function.*
→ *Negative thirty-two plus forty-eight equals sixteen, a positive value meaning the ball is moving upward.*

Answer: $v(1) = 16$ ft/s

Scan to watch

5. For the position function below, find the velocity at t = 3.

$s(t) = t^3 - 6t^2 + 9t$

→ *Take the first derivative of the position function.*
→ *The velocity function is three t squared minus twelve t plus nine.*
→ *Substitute t equals three into the velocity function.*
→ *Compute twenty-seven minus thirty-six plus nine to obtain zero, meaning the object is momentarily at rest.*

Answer: $v(3) = 0$

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6. Using the position function s(t) = t^3 - 6t^2 + 9t, find the acceleration at t = 3.

$s(t) = t^3 - 6t^2 + 9t, \quad t = 3$

→ *The first derivative gives the velocity function three t squared minus twelve t plus nine.*
→ *Take the derivative of the velocity function to get acceleration.*
→ *The acceleration function is six t minus twelve.*
→ *Substitute t equals three to get eighteen minus twelve, which equals six.*

Answer: $a(3) = 6$

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7. At t = 2 seconds, an object has velocity v = 5 and acceleration a = 3. Determine whether the object is speeding up or slowing down.

$v(2) = 5, \quad a(2) = 3$

→ *Identify the sign of the velocity, which is positive.*
→ *Identify the sign of the acceleration, which is also positive.*
→ *Since both velocity and acceleration have the same sign, the object is speeding up.*

Answer: Speeding up

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8. At t = 4 seconds, an object has velocity v = -8 and acceleration a = -2. Determine whether the object is speeding up or slowing down.

$v(4) = -8, \quad a(4) = -2$

→ *Identify the sign of the velocity, which is negative.*
→ *Identify the sign of the acceleration, which is also negative.*
→ *Since both velocity and acceleration share the same sign, the object is speeding up.*

Answer: Speeding up

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9. At t = 1 second, an object has velocity v = 6 and acceleration a = -4. Determine whether the object is speeding up or slowing down.

$v(1) = 6, \quad a(1) = -4$

→ *Identify the sign of the velocity, which is positive.*
→ *Identify the sign of the acceleration, which is negative.*
→ *Since velocity and acceleration have opposite signs, the object is slowing down.*

Answer: Slowing down

10. A particle moves along a line with position function below. Find the velocity function v(t).

$s(t) = 2t^3 - 9t^2 + 12t - 4$

→ *Differentiate each term of the position function with respect to t.*
→ *The derivative of two t cubed is six t squared.*
→ *The derivative of negative nine t squared is negative eighteen t.*
→ *The derivative of twelve t is twelve and the derivative of the constant is zero.*

Answer: $v(t) = 6t^2 - 18t + 12$