Related Rates: Advanced Applications

Calculus Worksheet · Grade 11-12

Name: ______________________ Date: ______________ Score: / 9

Learning Objectives

- Apply implicit differentiation with respect to time to geometric formulas
- Set up related rates problems using the Pythagorean theorem and other geometric relationships
- Solve for unknown rates of change by substituting given values

For each problem, draw a diagram, identify the given rate and the rate to find, write the equation, differentiate implicitly with respect to time, and solve.

1. A 10 ft ladder rests against a vertical wall. The bottom slides away from the wall at 1 ft/s. How fast is the top sliding down when the bottom is 6 ft from the wall?

$$x^2 + y^2 = 10^2, \quad \frac{dx}{dt} = 1$$

Answer: ______________________

2. A 13 ft ladder leans against a wall. The base slides away from the wall at 2 ft/s. How fast is the top descending when the base is 5 ft from the wall?

$$x^2 + y^2 = 13^2, \quad \frac{dx}{dt} = 2$$

Answer: ______________________

3. Air is pumped into a spherical balloon so that its volume increases at 100 cubic cm per second. How fast is the radius increasing when the radius is 25 cm?

$$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 100$$

Answer: ______________________

4. A circular oil slick expands so its radius grows at 0.5 m/s. How fast is the area increasing when the radius is 8 m?

$$A = \pi r^2, \quad \frac{dr}{dt} = 0.5$$

Answer: ______________________

5. A cube's edges expand at 3 cm/s. How fast is the volume increasing when the edge length is 10 cm?

$$V = s^3, \quad \frac{ds}{dt} = 3$$

Answer: ______________________

6. A 15 ft ladder slides down a wall. The top descends at 2 ft/s. How fast is the bottom moving away when the top is 9 ft above the ground?

$$x^2 + y^2 = 15^2, \quad \frac{dy}{dt} = -2$$

Answer: ______________________________________

7. Water is poured into a conical tank (radius 4 m, height 8 m) at 2 cubic m per minute. How fast is the water level rising when the depth is 3 m? Use the relation r = h/2.

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{12}$$

Answer: ______________________________________

8. Two cars leave the same point. Car A travels north at 60 mph and Car B travels east at 80 mph. How fast is the distance between them changing after 1 hour?

$$z^2 = x^2 + y^2, \quad \frac{dx}{dt} = 80, \quad \frac{dy}{dt} = 60$$

Answer: ______________________________________

9. Sand falls into a conical pile so its height equals its base radius. The volume increases at 10 cubic ft per minute. How fast is the height rising when the height is 5 ft?

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{3}$$

Answer: ______________________________________

Related Rates: Advanced Applications

Calculus Worksheet · Grade 11-12

ANSWER KEY & SOLUTIONS

Emphasize that students should differentiate before substituting numerical values, and remind them to memorize common geometric formulas like area of a circle and volume of a sphere.

Solutions

1. A 10 ft ladder rests against a vertical wall. The bottom slides away from the wall at 1 ft/s. How fast is the top sliding down when the bottom is 6 ft from the wall?

$$x^2 + y^2 = 10^2, \quad \frac{dx}{dt} = 1$$

→ *Draw a right triangle with the wall as the vertical leg and the ground as the horizontal leg.*
→ *Use the Pythagorean theorem: x squared plus y squared equals 100.*
→ *Differentiate both sides with respect to time to get 2x times dx/dt plus 2y times dy/dt equals zero.*
→ *When x equals 6, find y equals 8 from the Pythagorean theorem.*
→ *Substitute x equals 6, y equals 8, and dx/dt equals 1, then solve for dy/dt to get negative three-fourths ft per second.*

Answer: $$\frac{dy}{dt} = -\frac{3}{4}\text{ ft/s}$$

---

2. A 13 ft ladder leans against a wall. The base slides away from the wall at 2 ft/s. How fast is the top descending when the base is 5 ft from the wall?

$$x^2 + y^2 = 13^2, \quad \frac{dx}{dt} = 2$$

→ *Set up the Pythagorean relationship with hypotenuse 13.*
→ *Differentiate implicitly with respect to time.*
→ *When x equals 5, solve for y to get 12.*
→ *Substitute the known values into the differentiated equation.*
→ *Solve for dy/dt to obtain negative five-sixths ft per second.*

Answer: $$\frac{dy}{dt} = -\frac{5}{6}\text{ ft/s}$$

---

3. Air is pumped into a spherical balloon so that its volume increases at 100 cubic cm per second. How fast is the radius increasing when the radius is 25 cm?

$$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 100$$

→ *Write the volume formula for a sphere.*
→ *Differentiate both sides with respect to time to get dV/dt equals 4 pi r squared times dr/dt.*
→ *Substitute dV/dt equals 100 and r equals 25.*
→ *Solve for dr/dt to get 1 over 25 pi cm per second.*

Answer: $$\frac{dr}{dt} = \frac{1}{25\pi}\text{ cm/s}$$

---

4. A circular oil slick expands so its radius grows at 0.5 m/s. How fast is the area increasing when the radius is 8 m?

$A = \pi r^2, \quad \frac{dr}{dt} = 0.5$

→ *Use the area formula for a circle.*
→ *Differentiate with respect to time to get dA/dt equals 2 pi r times dr/dt.*
→ *Substitute r equals 8 and dr/dt equals one-half.*
→ *Compute dA/dt equals 8 pi square meters per second.*

Answer: $\frac{dA}{dt} = 8\pi \text{ m}^2/\text{s}$

---

5. A cube's edges expand at 3 cm/s. How fast is the volume increasing when the edge length is 10 cm?

$V = s^3, \quad \frac{ds}{dt} = 3$

→ *Write the cube volume formula V equals s cubed.*
→ *Differentiate to get dV/dt equals 3 s squared times ds/dt.*
→ *Substitute s equals 10 and ds/dt equals 3.*
→ *Compute dV/dt equals 900 cubic cm per second.*

Answer: $\frac{dV}{dt} = 900 \text{ cm}^3/\text{s}$

---

6. A 15 ft ladder slides down a wall. The top descends at 2 ft/s. How fast is the bottom moving away when the top is 9 ft above the ground?

$x^2 + y^2 = 15^2, \quad \frac{dy}{dt} = -2$

→ *Apply the Pythagorean theorem with hypotenuse 15.*
→ *Differentiate implicitly to get 2x dx/dt plus 2y dy/dt equals zero.*
→ *When y equals 9, solve for x to get 12.*
→ *Substitute the values and solve for dx/dt to obtain three-halves ft per second.*

Answer: $\frac{dx}{dt} = \frac{3}{2} \text{ ft/s}$

---

7. Water is poured into a conical tank (radius 4 m, height 8 m) at 2 cubic m per minute. How fast is the water level rising when the depth is 3 m? Use the relation r = h/2.

$V = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{12}$

→ *Express r in terms of h using similar triangles: r equals h over 2.*
→ *Substitute into the cone volume formula to get V equals pi h cubed over 12.*
→ *Differentiate with respect to time to get dV/dt equals pi h squared over 4 times dh/dt.*
→ *Substitute dV/dt equals 2 and h equals 3, then solve for dh/dt to get 8 over 9 pi m per min.*

Answer: $\frac{dh}{dt} = \frac{8}{9\pi} \text{ m/min}$

---

8. Two cars leave the same point. Car A travels north at 60 mph and Car B travels east at 80 mph. How fast is the distance between them changing after 1 hour?

$$z^2 = x^2 + y^2, \quad \frac{dx}{dt} = 80, \quad \frac{dy}{dt} = 60$$

→ *Set up the Pythagorean relationship for the distance z between the cars.*
→ *Differentiate implicitly to get 2z dz/dt equals 2x dx/dt plus 2y dy/dt.*
→ *After 1 hour, x equals 80, y equals 60, so z equals 100.*
→ *Substitute the values and solve for dz/dt to get 100 mph.*

Answer: $\frac{dz}{dt} = 100 \text{ mph}$

---

9. Sand falls into a conical pile so its height equals its base radius. The volume increases at 10 cubic ft per minute. How fast is the height rising when the height is 5 ft?

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{3}$$

→ *Since r equals h, substitute into the cone volume formula to get V equals pi h cubed over 3.*
→ *Differentiate with respect to time to get dV/dt equals pi h squared times dh/dt.*
→ *Substitute dV/dt equals 10 and h equals 5.*
→ *Solve for dh/dt to obtain 2 over 5 pi ft per minute.*

Answer: $\frac{dh}{dt} = \frac{2}{5\pi} \text{ ft/min}$