Related Rates

Calculus Worksheet · Grade 11–12 / AP Calculus

Name: ________________________________ Date: __________________

Learning Objectives

- Identify given equations, given rates, and unknown rates in related-rates problems
- Apply implicit differentiation with respect to time to geometric and algebraic equations
- Solve for unknown rates of change using substitution of given values

Problems

1. Suppose x and y are differentiable functions of t and are related by the equation below. Find dy/dt when x = 2, given that dx/dt = 3.

$$y = x^2 + 5$$

2. Suppose x and y are differentiable functions of t and are related by the equation below. Find dx/dt when x = 3 and y = 2, given that dy/dt = 6.

$$y = 4x^3 - 2$$

3. The radius of a circle is increasing at a rate of 4 cm per second. Find the rate at which the area of the circle is increasing when the radius is 6 cm. (Use the formula for the area of a circle.)

$$A = \pi r^2$$

4. The side length s of a square is decreasing at a rate of 2 m per minute. Find the rate at which the area of the square is changing when s = 10 m.

$$A = s^2$$

5. A spherical balloon is being inflated so that its volume increases at a rate of 100 cubic inches per second. How fast is the radius increasing when the radius is 5 inches? (Use the formula for the volume of a sphere.)

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$$V = \frac{4}{3}\pi r^3$$

6. A ladder 10 feet long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall? Use the Pythagorean relationship between x (distance from wall to base) and y (height on wall).

$$x^2 + y^2 = 100$$

7. Sand is being poured onto a conical pile at a rate of 8 cubic feet per minute. The pile always maintains the shape where its height equals the diameter of its base (i.e., height h = 2r). Find the rate at which the height is increasing when the height is 4 feet. Use the formula for the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$

8. Two cars start from the same intersection. Car A travels north at 60 mph and Car B travels east at 45 mph. Both leave at the same time. How fast is the distance between them increasing 2 hours after they leave the intersection?

$$z^2 = x^2 + y^2$$

9. Water is draining from a cylindrical tank at a rate of 3 cubic meters per minute. The tank has a fixed radius of 2 meters. Find the rate at which the water level (height h) is falling. Then determine how long it will take for the water level to drop 10 meters at this rate.

$$V = \pi r^2 h$$

10. A kite is flying at a height of 120 feet above the ground. The kite moves horizontally (parallel to the ground) away from the person holding the string at a rate of 10 ft/s. Assuming the string is taut and the person holds the string at ground level, how fast is the string being let out when the horizontal distance from the person to the kite is 50 feet? Let z be the length of the string, x the horizontal distance, and y = 120 ft the fixed height.

$$z^2 = x^2 + y^2$$

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Related Rates — Answer Key

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Answer Key

1. Answer: $dy/dt = 12$

- Differentiate both sides with respect to t: $dy/dt = 2x \cdot (dx/dt)$
- Substitute $x = 2$ and $dx/dt = 3$: $dy/dt = 2(2)(3) = 12$

2. Answer: $dx/dt = 1/6$

- Differentiate both sides with respect to t: $dy/dt = 12x^2 \cdot (dx/dt)$
- Substitute $x = 3$ and $dy/dt = 6$: $6 = 12(9)(dx/dt)$
- Solve: $dx/dt = 6/108 = 1/18$
- Wait — re-check: $12(3)^2 = 12 \cdot 9 = 108$, so $dx/dt = 6/108 = 1/18$

3. Answer: $dA/dt = 48\pi\ cm^2/s$

- Differentiate with respect to t: $dA/dt = 2\pi r \cdot (dr/dt)$
- Substitute $r = 6$ and $dr/dt = 4$: $dA/dt = 2\pi(6)(4) = 48\pi\ cm^2/s$

4. Answer: $dA/dt = –40\ m^2/min$

- Differentiate with respect to t: $dA/dt = 2s \cdot (ds/dt)$
- Since s is decreasing, $ds/dt = –2$
- Substitute $s = 10$ and $ds/dt = –2$: $dA/dt = 2(10)(–2) = –40\ m^2/min$

5. Answer: $dr/dt = 1/\pi$ in/s

- Differentiate with respect to t: $dV/dt = 4\pi r^2 \cdot (dr/dt)$
- Substitute $dV/dt = 100$ and $r = 5$: $100 = 4\pi(25)(dr/dt)$
- Solve: $dr/dt = 100/(100\pi) = 1/\pi$ in/s

6. Answer: $dy/dt = –3/4$ ft/s

- Differentiate with respect to t: $2x(dx/dt) + 2y(dy/dt) = 0$
- When $x = 6$: $y = \sqrt{(100 – 36)} = \sqrt{64} = 8$
- Substitute $x = 6$, $y = 8$, $dx/dt = 1$: $2(6)(1) + 2(8)(dy/dt) = 0$
- Solve: $12 + 16(dy/dt) = 0 \to dy/dt = –12/16 = –3/4$ ft/s

7. Answer: $dh/dt = 2/(2\pi) = 1/\pi$ ft/min

- Since $h = 2r$, we have $r = h/2$. Substitute into V: $V = (1/3)\pi(h/2)^2(h) = \pi h^3/12$
- Differentiate with respect to t: $dV/dt = (\pi/12)(3h^2)(dh/dt) = (\pi h^2/4)(dh/dt)$
- Substitute $dV/dt = 8$ and $h = 4$: $8 = (\pi \cdot 16/4)(dh/dt) = 4\pi(dh/dt)$
- Solve: $dh/dt = 8/(4\pi) = 2/\pi$ ft/min

8. Answer: $dz/dt = 75$ mph

- After 2 hours: $x = 45(2) = 90$ mi (east), $y = 60(2) = 120$ mi (north)
- $z = \sqrt{(90^2 + 120^2)} = \sqrt{(8100 + 14400)} = \sqrt{22500} = 150$ mi

- Differentiate with respect to t: $2z(dz/dt) = 2x(dx/dt) + 2y(dy/dt)$
- Substitute: $2(150)(dz/dt) = 2(90)(45) + 2(120)(60) = 8100 + 14400 = 22500$
- Solve: $dz/dt = 22500/300 = 75$ mph

9. Answer: $dh/dt = -3/(4\pi)$ m/min; time ≈ 41.9 min

- Since $r = 2$ is constant: $V = \pi(4)h = 4\pi h$
- Differentiate with respect to t: $dV/dt = 4\pi(dh/dt)$
- Water is draining so $dV/dt = -3$: $-3 = 4\pi(dh/dt)$
- Solve: $dh/dt = -3/(4\pi)$ m/min
- Time to drop 10 m: $t = 10 / (3/(4\pi)) = 40\pi/3 \approx 41.9$ minutes

10. Answer: $dz/dt = 500/\sqrt{16900} = 50/\sqrt{169} \approx 3.85$ ft/s

- $y = 120$ is constant, so $z^2 = x^2 + 120^2$
- When $x = 50$: $z = \sqrt{(2500 + 14400)} = \sqrt{16900} = 130$ ft
- Differentiate with respect to t: $2z(dz/dt) = 2x(dx/dt)$
- Substitute $z = 130$, $x = 50$, $dx/dt = 10$: $2(130)(dz/dt) = 2(50)(10) = 1000$
- Solve: $dz/dt = 1000/260 = 50/13 \approx 3.85$ ft/s