Related Rates: Geometric Applications

Calculus Worksheet · Grade 11–12

Name: _____ Date: _____

Learning Objectives

- Identify given rates and unknown rates in related rates word problems
- Apply geometric formulas (Pythagorean theorem, area, volume) and differentiate implicitly with respect to time
- Substitute known values to solve for unknown rates of change

Problems

1. A ladder 10 ft long leans against a vertical wall. If the bottom of the ladder slides away from the wall at 1 ft/sec, find dy/dt when x = 6 ft. (y is the height on the wall.)

$$x^2 + y^2 = 100$$

2. The area of a square is increasing at 8 sq cm/sec. How fast is the side length increasing when the side is 4 cm long?

$$A = s^2$$

3. The radius of a circle is increasing at 3 cm/sec. How fast is the area of the circle increasing when the radius is 5 cm?

$$A = \pi r^2$$

4. A ladder 13 ft long rests against a vertical wall. The bottom slides away from the wall at 2 ft/sec. How fast is the top of the ladder sliding down the wall when the bottom is 5 ft from the wall?

$$x^2 + y^2 = 169$$

5. The volume of a sphere is increasing at 100 cubic cm/sec. How fast is the radius increasing when the radius is 5 cm?

$$V = \frac{4}{3}\pi r^3$$

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6. Water is draining from a cylindrical tank of radius 4 ft at 2 cubic ft/min. How fast is the water level dropping?

$$V = \pi r^2 h$$

7. A ladder 17 ft long leans against a wall. The top slides down the wall at 2 ft/sec. How fast is the bottom of the ladder sliding away from the wall when the bottom is 8 ft from the wall?

$$x^2 + y^2 = 289$$

8. The surface area of a sphere is decreasing at 4 sq cm/sec. How fast is the radius changing when the radius is 3 cm?

$$S = 4\pi r^2$$

9. A conical water tank (vertex down) has a radius of 6 ft and height of 12 ft. Water flows in at 4 cubic ft/min. How fast is the water level rising when the water is 4 ft deep? (Hint: use similar triangles to relate radius to height before differentiating.)

$$V = \frac{1}{3}\pi r^2 h$$

10. Two cars start from the same point at the same time. Car A travels east at 60 mph and Car B travels north at 80 mph. How fast is the distance between the two cars increasing 1.5 hours after they start?

$$D^2 = x^2 + y^2$$

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Related Rates: Geometric Applications — Answer Key

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Answer Key

1. Answer: dy/dt = –3/4 ft/sec

- When x = 6, use Pythagorean theorem: $6^2 + y^2 = 100 \rightarrow y = 8$
- Differentiate both sides with respect to t: $2x(dx/dt) + 2y(dy/dt) = 0$
- Substitute x = 6, y = 8, dx/dt = 1: $2(6)(1) + 2(8)(dy/dt) = 0$
- $12 + 16(dy/dt) = 0 \rightarrow dy/dt = –12/16 = –3/4$ ft/sec

2. Answer: ds/dt = 1 cm/sec

- Differentiate $A = s^2$ with respect to t: $dA/dt = 2s(ds/dt)$
- Substitute dA/dt = 8 and s = 4: $8 = 2(4)(ds/dt)$
- $8 = 8(ds/dt) \rightarrow ds/dt = 1$ cm/sec

3. Answer: dA/dt = 30π cm²/sec

- Differentiate $A = \pi r^2$ with respect to t: $dA/dt = 2\pi r(dr/dt)$
- Substitute r = 5 and dr/dt = 3: $dA/dt = 2\pi(5)(3)$
- $dA/dt = 30\pi$ cm²/sec

4. Answer: dy/dt = –5/6 ft/sec

- When x = 5: $5^2 + y^2 = 169 \rightarrow y^2 = 144 \rightarrow y = 12$
- Differentiate: $2x(dx/dt) + 2y(dy/dt) = 0$
- Substitute x = 5, y = 12, dx/dt = 2: $2(5)(2) + 2(12)(dy/dt) = 0$
- $20 + 24(dy/dt) = 0 \rightarrow dy/dt = –20/24 = –5/6$ ft/sec

5. Answer: dr/dt = 1/π cm/sec

- Differentiate $V = (4/3)\pi r^3$ with respect to t: $dV/dt = 4\pi r^2(dr/dt)$
- Substitute dV/dt = 100 and r = 5: $100 = 4\pi(25)(dr/dt)$
- $100 = 100\pi(dr/dt) \rightarrow dr/dt = 100/(100\pi) = 1/\pi$ cm/sec

6. Answer: dh/dt = –1/(8π) ft/min

- Since radius is constant at 4 ft: $V = \pi(4^2)h = 16\pi h$
- Differentiate with respect to t: $dV/dt = 16\pi(dh/dt)$
- Water is draining so dV/dt = –2: $–2 = 16\pi(dh/dt)$
- $dh/dt = –2/(16\pi) = –1/(8\pi)$ ft/min

7. Answer: dx/dt = 15/8 ft/sec

- When x = 8: $64 + y^2 = 289 \rightarrow y^2 = 225 \rightarrow y = 15$
- Differentiate: $2x(dx/dt) + 2y(dy/dt) = 0$
- Substitute x = 8, y = 15, dy/dt = –2: $2(8)(dx/dt) + 2(15)(–2) = 0$

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- $16(dx/dt) - 60 = 0 \rightarrow dx/dt = 60/16 = 15/8$ ft/sec

8. Answer: $dr/dt = -1/(6\pi)$ cm/sec

- Differentiate $S = 4\pi r^2$ with respect to t: $dS/dt = 8\pi r(dr/dt)$
- Surface area is decreasing so $dS/dt = -4$; substitute $r = 3$: $-4 = 8\pi(3)(dr/dt)$
- $-4 = 24\pi(dr/dt) \rightarrow dr/dt = -4/(24\pi) = -1/(6\pi)$ cm/sec

9. Answer: $dh/dt = 1/\pi$ ft/min

- By similar triangles: $r/h = 6/12 = 1/2$, so $r = h/2$
- Substitute into volume formula: $V = (1/3)\pi(h/2)^2h = (1/3)\pi(h^2/4)h = \pi h^3/12$
- Differentiate with respect to t: $dV/dt = (\pi/12)(3h^2)(dh/dt) = (\pi h^2/4)(dh/dt)$
- Substitute $dV/dt = 4$ and $h = 4$: $4 = (\pi \cdot 16/4)(dh/dt) = 4\pi(dh/dt)$
- $dh/dt = 4/(4\pi) = 1/\pi$ ft/min

10. Answer: dD/dt = 100 mph

- After 1.5 hours: $x = 60(1.5) = 90$ miles, $y = 80(1.5) = 120$ miles
- Find D: $D^2 = 90^2 + 120^2 = 8100 + 14400 = 22500 \rightarrow D = 150$ miles
- Differentiate $D^2 = x^2 + y^2$ with respect to t: $2D(dD/dt) = 2x(dx/dt) + 2y(dy/dt)$
- Substitute $D = 150$, $x = 90$, $y = 120$, $dx/dt = 60$, $dy/dt = 80$:
- $2(150)(dD/dt) = 2(90)(60) + 2(120)(80) = 10800 + 19200 = 30000$
- $300(dD/dt) = 30000 \rightarrow dD/dt = 100$ mph

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