Related Rates with Implicit Differentiation

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 9

Learning Objectives

- Apply implicit differentiation with respect to time to geometric formulas
- Use the product rule when differentiating related rates expressions
- Solve for an unknown rate by substituting given values into a derived equation

For each problem, differentiate the given formula implicitly with respect to time, then substitute the given values to find the requested rate.

1. Given the area of a circle, find dA/dt when r=2 and dr/dt=3.

$$A = \pi r^2,\ r = 2,\ \frac{dr}{dt} = 3$$

Answer: _____

2. Given the lateral surface area of a cylinder, find dr/dt when r=2, h=4, dA/dt=16π, and dh/dt=2.

$$A = 2\pi rh,\ r = 2,\ h = 4,\ \frac{dA}{dt} = 16\pi,\ \frac{dh}{dt} = 2$$

Answer: _____

3. The radius of a circle is increasing at 5 cm/s. Find the rate of change of the area when r=10 cm.

$$A = \pi r^2,\ r = 10,\ \frac{dr}{dt} = 5$$

Answer: _____

4. The volume of a sphere V = (4/3)π r^3 is changing. Find dV/dt when r=3 and dr/dt=2.

$$V = \frac{4}{3}\pi r^3,\ r = 3,\ \frac{dr}{dt} = 2$$

Answer: _____

5. For a square with side s, find dA/dt when s=6 and ds/dt=4.

$$A = s^2,\ s = 6,\ \frac{ds}{dt} = 4$$

Answer: _____

6. A cylinder has volume V = π r^2 h. Find dV/dt when r=2, h=5, dr/dt=1, and dh/dt=3.

$$V = \pi r^2 h,\ r = 2,\ h = 5,\ \frac{dr}{dt} = 1,\ \frac{dh}{dt} = 3$$

Answer: _____

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7. A right triangle has legs x and y with x^2 + y^2 = z^2. Find dz/dt when x=3, y=4, dx/dt=2, and dy/dt=1.

$$x^2 + y^2 = z^2,\ x = 3,\ y = 4,\ \frac{dx}{dt} = 2,\ \frac{dy}{dt} = 1$$

Answer: _____

8. The area of a rectangle is A=lw. Find dA/dt when l=8, w=5, dl/dt=3, and dw/dt=2.

$$A = lw,\ l = 8,\ w = 5,\ \frac{dl}{dt} = 3,\ \frac{dw}{dt} = 2$$

Answer: _____

9. The surface area of a sphere is S=4π r^2. Find dS/dt when r=4 and dr/dt=2.

$$S = 4\pi r^2,\ r = 4,\ \frac{dr}{dt} = 2$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students to differentiate first symbolically before substituting numerical values, and to apply the product rule when two variables both change with time.

Solutions

1. Given the area of a circle, find dA/dt when r=2 and dr/dt=3.

$A = \pi r^2,\ r = 2,\ \frac{dr}{dt} = 3$

→ *Differentiate both sides with respect to time, treating pi as a constant.*
→ *The derivative of r squared with respect to time is two r times dr/dt.*
→ *Substitute r equals two and dr/dt equals three into the equation.*
→ *Multiply two times two times three times pi to get twelve pi.*

Answer: $\frac{dA}{dt} = 12\pi$

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2. Given the lateral surface area of a cylinder, find dr/dt when r=2, h=4, dA/dt=16π, and dh/dt=2.

$A = 2\pi rh,\ r = 2,\ h = 4,\ \frac{dA}{dt} = 16\pi,\ \frac{dh}{dt} = 2$

→ *Differentiate A equals two pi r h with respect to time using the product rule.*
→ *Obtain dA/dt equals two pi h dr/dt plus two pi r dh/dt.*
→ *Substitute dA/dt equals sixteen pi, h equals four, r equals two, and dh/dt equals two.*
→ *Simplify to sixteen pi equals eight pi dr/dt plus eight pi.*
→ *Subtract eight pi from both sides and divide by eight pi to get dr/dt equals one.*

Answer: $\frac{dr}{dt} = 1$

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3. The radius of a circle is increasing at 5 cm/s. Find the rate of change of the area when r=10 cm.

$A = \pi r^2,\ r = 10,\ \frac{dr}{dt} = 5$

→ *Differentiate area with respect to time to get dA/dt equals two pi r dr/dt.*
→ *Substitute r equals ten and dr/dt equals five.*
→ *Multiply two times ten times five times pi.*
→ *The result is one hundred pi square centimeters per second.*

Answer: $\frac{dA}{dt} = 100\pi\ \text{cm}^2/\text{s}$

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4. The volume of a sphere V = (4/3)π r^3 is changing. Find dV/dt when r=3 and dr/dt=2.

$V = \frac{4}{3}\pi r^3,\ r = 3,\ \frac{dr}{dt} = 2$

→ *Differentiate V with respect to time to obtain dV/dt equals four pi r squared dr/dt.*
→ *Substitute r equals three and dr/dt equals two.*
→ *Compute four times nine times two to get seventy-two.*
→ *Therefore dV/dt equals seventy-two pi.*

Answer: $\frac{dV}{dt} = 72\pi$

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5. For a square with side s, find dA/dt when s=6 and ds/dt=4.

$$A = s^2,\ s = 6,\ \frac{ds}{dt} = 4$$

→ *Differentiate area equals s squared with respect to time.*
→ *Obtain dA/dt equals two s ds/dt.*
→ *Substitute s equals six and ds/dt equals four.*
→ *Multiply two times six times four to get forty-eight.*

Answer: $\frac{dA}{dt} = 48$

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6. A cylinder has volume V = π r^2 h. Find dV/dt when r=2, h=5, dr/dt=1, and dh/dt=3.

$$V = \pi r^2 h,\ r = 2,\ h = 5,\ \frac{dr}{dt} = 1,\ \frac{dh}{dt} = 3$$

→ *Differentiate V with respect to time using the product rule.*
→ *Get dV/dt equals two pi r h dr/dt plus pi r squared dh/dt.*
→ *Substitute r equals two, h equals five, dr/dt equals one, and dh/dt equals three.*
→ *Compute twenty pi plus twelve pi to obtain thirty-two pi.*

Answer: $\frac{dV}{dt} = 32\pi$

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7. A right triangle has legs x and y with x^2 + y^2 = z^2. Find dz/dt when x=3, y=4, dx/dt=2, and dy/dt=1.

$$x^2 + y^2 = z^2,\ x = 3,\ y = 4,\ \frac{dx}{dt} = 2,\ \frac{dy}{dt} = 1$$

→ *Differentiate both sides with respect to time.*
→ *Obtain two x dx/dt plus two y dy/dt equals two z dz/dt.*
→ *Find z from three squared plus four squared equals twenty-five, so z equals five.*
→ *Substitute values to get six times two plus eight times one equals ten dz/dt.*
→ *Solve to get dz/dt equals two.*

Answer: $\frac{dz}{dt} = 2$

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8. The area of a rectangle is A=lw. Find dA/dt when l=8, w=5, dl/dt=3, and dw/dt=2.

$$A = lw,\ l = 8,\ w = 5,\ \frac{dl}{dt} = 3,\ \frac{dw}{dt} = 2$$

→ *Differentiate A equals lw with respect to time using the product rule.*
→ *Obtain dA/dt equals dl/dt times w plus l times dw/dt.*
→ *Substitute l equals eight, w equals five, dl/dt equals three, and dw/dt equals two.*
→ *Compute three times five plus eight times two equals fifteen plus sixteen, giving thirty-one.*

Answer: $\frac{dA}{dt} = 31$

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9. The surface area of a sphere is S=4π r^2. Find dS/dt when r=4 and dr/dt=2.

$$S = 4\pi r^2,\ r = 4,\ \frac{dr}{dt} = 2$$

→ *Differentiate S equals four pi r squared with respect to time.*
→ *Obtain dS/dt equals eight pi r dr/dt.*
→ *Substitute r equals four and dr/dt equals two.*
→ *Compute eight times four times two times pi to get sixty-four pi.*

Answer: $\frac{dS}{dt} = 64\pi$