Related Rates Word Problems

Calculus Worksheet · Grade 11-12

Name: ______________________ Date: ______________ Score: / 10

Learning Objectives

- Identify the given equation, given rate, and unknown rate in related rates problems
- Apply implicit differentiation with respect to time to relate changing variables
- Use geometric formulas to set up and solve real-world related rates problems

Read each problem carefully, identify the given equation and rates, then use implicit differentiation with respect to time to find the requested rate of change.

1. Suppose x and y are differentiable functions of t related by the equation below. Find dy/dt when x=1, given that dx/dt = 2.

$$y = x^2 + 3, \quad \frac{dx}{dt} = 2, \; x = 1$$

Answer: ______________________

2. Suppose x and y are differentiable functions of t related by the equation below. Find dx/dt when y=2, given that dy/dt = 5.

$$y^2 = x + 7, \quad \frac{dy}{dt} = 5, \; y = 2$$

Answer: ______________________

3. The radius of a circle is increasing at a rate of 3 cm/s. How fast is the area of the circle changing when the radius is 10 cm?

$$A = \pi r^2, \quad \frac{dr}{dt} = 3, \; r = 10$$

Answer: ______________________

4. Air is pumped into a spherical balloon so that its volume increases at 100 cubic centimeters per second. How fast is the radius increasing when the radius is 5 cm?

$$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 100, \; r = 5$$

Answer: ______________________

5. Each side of a square is increasing at a rate of 6 cm/s. How fast is the area of the square increasing when each side is 12 cm?

$$A = s^2, \quad \frac{ds}{dt} = 6, \; s = 12$$

Answer: ______________________

6. The edge of a cube is increasing at 2 inches per minute. How fast is the volume of the cube increasing when the edge is 4 inches?

$$V = s^3, \quad \frac{ds}{dt} = 2, \ s = 4$$

Answer: ____

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7. A 13-foot ladder leans against a wall. The bottom slides away from the wall at 2 ft/s. How fast is the top sliding down when the bottom is 5 feet from the wall?

$$x^2 + y^2 = 13^2, \quad \frac{dx}{dt} = 2, \ x = 5$$

Answer: ____

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8. Water is poured into a right circular cone with radius 4 m and height 8 m at 2 cubic meters per minute. Using r = h/2, how fast is the water level rising when the water is 3 m deep?

$$V = \frac{1}{3}\pi r^2 h, \ r = \frac{h}{2}, \ \frac{dV}{dt} = 2, \ h = 3$$

Answer: ____

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9. The radius of a circular oil spill is increasing at 0.5 m/s. How fast is the circumference increasing when the radius is 20 m?

$$C = 2\pi r, \quad \frac{dr}{dt} = 0.5, \ r = 20$$

Answer: ____

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10. The area of a square is increasing at 50 cm^2/s. How fast is the side length increasing when each side is 10 cm?

$$A = s^2, \quad \frac{dA}{dt} = 50, \ s = 10$$

Answer: ____

Encourage students to write down the given equation, the given rate, and the unknown rate before differentiating; remind them to substitute values only after differentiating.

Solutions

1. Suppose x and y are differentiable functions of t related by the equation below. Find dy/dt when x=1, given that dx/dt = 2.

$y = x^2 + 3, \quad \frac{dx}{dt} = 2, \; x = 1$

→ *Differentiate both sides of the equation with respect to t.*
→ *The derivative of y is dy/dt and the derivative of x squared is 2x times dx/dt.*
→ *Substitute x equals 1 and dx/dt equals 2 into the differentiated equation.*
→ *Simplify to obtain dy/dt equals 4.*

Answer: $\frac{dy}{dt} = 4$

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2. Suppose x and y are differentiable functions of t related by the equation below. Find dx/dt when y=2, given that dy/dt = 5.

$y^2 = x + 7, \quad \frac{dy}{dt} = 5, \; y = 2$

→ *Differentiate both sides of the equation with respect to t.*
→ *The derivative of y squared is 2y times dy/dt and the derivative of x is dx/dt.*
→ *Substitute y equals 2 and dy/dt equals 5.*
→ *Solve for dx/dt to get 20.*

Answer: $\frac{dx}{dt} = 20$

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3. The radius of a circle is increasing at a rate of 3 cm/s. How fast is the area of the circle changing when the radius is 10 cm?

$A = \pi r^2, \quad \frac{dr}{dt} = 3, \; r = 10$

→ *Write the area formula for a circle.*
→ *Differentiate both sides with respect to time to get dA/dt equals 2 pi r times dr/dt.*
→ *Substitute r equals 10 and dr/dt equals 3.*
→ *Simplify to obtain 60 pi square centimeters per second.*

Answer: $\frac{dA}{dt} = 60\pi \text{ cm}^2/\text{s}$

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4. Air is pumped into a spherical balloon so that its volume increases at 100 cubic centimeters per second. How fast is the radius increasing when the radius is 5 cm?

$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 100, \; r = 5$

→ *Write the volume formula for a sphere.*
→ *Differentiate both sides with respect to time to get dV/dt equals 4 pi r squared times dr/dt.*
→ *Substitute r equals 5 and dV/dt equals 100.*
→ *Solve for dr/dt to obtain one over pi centimeters per second.*

Answer: $\frac{dr}{dt} = \frac{1}{\pi}$ cm/s

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5. Each side of a square is increasing at a rate of 6 cm/s. How fast is the area of the square increasing when each side is 12 cm?

$A = s^2, \quad \frac{ds}{dt} = 6, \; s = 12$

→ *Write the area formula for a square in terms of side length s.*
→ *Differentiate both sides with respect to time to get dA/dt equals 2s times ds/dt.*
→ *Substitute s equals 12 and ds/dt equals 6.*
→ *Simplify to obtain 144 square centimeters per second.*

Answer: $\frac{dA}{dt} = 144 \text{ cm}^2/\text{s}$

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6. The edge of a cube is increasing at 2 inches per minute. How fast is the volume of the cube increasing when the edge is 4 inches?

$V = s^3, \quad \frac{ds}{dt} = 2, \; s = 4$

→ *Write the volume formula for a cube in terms of edge length s.*
→ *Differentiate both sides with respect to time to get dV/dt equals 3 s squared times ds/dt.*
→ *Substitute s equals 4 and ds/dt equals 2.*
→ *Simplify to obtain 96 cubic inches per minute.*

Answer: $\frac{dV}{dt} = 96 \text{ in}^3/\text{min}$

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7. A 13-foot ladder leans against a wall. The bottom slides away from the wall at 2 ft/s. How fast is the top sliding down when the bottom is 5 feet from the wall?

$x^2 + y^2 = 13^2, \quad \frac{dx}{dt} = 2, \; x = 5$

→ *Use the Pythagorean theorem to relate x and y on the ladder.*
→ *Differentiate both sides with respect to time to obtain 2x times dx/dt plus 2y times dy/dt equals 0.*
→ *Find y using the Pythagorean theorem when x equals 5, giving y equals 12.*
→ *Substitute the known values and solve for dy/dt to obtain negative five sixths feet per second.*

Answer: $\frac{dy}{dt} = -\frac{5}{6}$ ft/s

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8. Water is poured into a right circular cone with radius 4 m and height 8 m at 2 cubic meters per minute. Using r = h/2, how fast is the water level rising when the water is 3 m deep?

$$V = \frac{1}{3}\pi r^2 h,\ r = \frac{h}{2},\ \frac{dV}{dt} = 2,\ h = 3$$

→ *Substitute r equals h over 2 into the cone volume formula to express V in terms of h only.*
→ *Simplify to get V equals pi over twelve times h cubed.*
→ *Differentiate both sides with respect to time to get dV/dt equals pi over four times h squared times dh/dt.*
→ *Substitute h equals 3 and dV/dt equals 2 and solve for dh/dt to obtain 8 over 9 pi meters per minute.*

Answer: $\frac{dh}{dt} = \frac{8}{9\pi}$ m/min

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9. The radius of a circular oil spill is increasing at 0.5 m/s. How fast is the circumference increasing when the radius is 20 m?

$$C = 2\pi r,\quad \frac{dr}{dt} = 0.5,\ r = 20$$

→ *Write the circumference formula for a circle.*
→ *Differentiate both sides with respect to time to get dC/dt equals 2 pi times dr/dt.*
→ *Substitute dr/dt equals one half.*
→ *Simplify to obtain pi meters per second.*

Answer: $\frac{dC}{dt} = \pi$ m/s

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10. The area of a square is increasing at 50 cm^2/s. How fast is the side length increasing when each side is 10 cm?

$$A = s^2,\quad \frac{dA}{dt} = 50,\ s = 10$$

→ *Write the area formula for a square in terms of side length s.*
→ *Differentiate both sides with respect to time to get dA/dt equals 2s times ds/dt.*
→ *Substitute s equals 10 and dA/dt equals 50.*
→ *Solve for ds/dt to obtain 2.5 centimeters per second.*

Answer: $\frac{ds}{dt} = 2.5$ cm/s