Rolle's Theorem

Calculus Worksheet · Grade 11-12

Name: ______________________ Date: ______________ Score: / 10

Learning Objectives

- Verify the three conditions of Rolle's Theorem for a given function on a closed interval
- Find the value(s) of c guaranteed by Rolle's Theorem
- Distinguish between Rolle's Theorem and the Mean Value Theorem

For each function, verify that the three conditions of Rolle's Theorem are satisfied on the given interval, then find all values of c in the open interval where f'(c) = 0.

1. Verify Rolle's Theorem and find c for the function on the closed interval [0, 3].

$$f(x) = x^3 - x^2 - 6x + 2$$

Answer: ______________________

2. Verify Rolle's Theorem and find c for the function on the closed interval [-2, 2].

$$f(x) = x^2 - 4$$

Answer: ______________________

3. Verify Rolle's Theorem and find c for the function on the closed interval [0, 2\pi].

$$f(x) = \sin(x)$$

Answer: ______________________

4. Verify Rolle's Theorem and find c for the function on the closed interval [1, 3].

$$f(x) = x^2 - 4x + 3$$

Answer: ______________________

5. Verify Rolle's Theorem and find c for the function on the closed interval [-1, 1].

$$f(x) = x^3 - x$$

Answer: ______________________

6. Verify Rolle's Theorem and find c for the function on the closed interval [0, 4].

$$f(x) = x^2 - 4x + 1$$

Answer: ______________________

7. Verify Rolle's Theorem and find c for the function on the closed interval [-3, 3].

$$f(x) = 9 - x^2$$

Answer: ______________________

8. Verify Rolle's Theorem and find c for the function on the closed interval [0, \pi].

$f(x) = \sin^2(x)$

Answer: ________________________________

9. Verify Rolle's Theorem and find c for the function on the closed interval [-2, 2].

$f(x) = x^4 - 2x^2$

Answer: ________________________________

10. Verify Rolle's Theorem and find c for the function on the closed interval [1, 5].

$f(x) = x^2 - 6x + 5$

Answer: ________________________________

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ANSWER KEY & SOLUTIONS

Remind students that Rolle's Theorem requires continuity on [a,b], differentiability on (a,b), and f(a) = f(b); only then does a c exist with f'(c) = 0.

Solutions

1. Verify Rolle's Theorem and find c for the function on the closed interval [0, 3].

$$f(x) = x^3 - x^2 - 6x + 2$$

→ *The function is a polynomial, so it is continuous on [0, 3] and differentiable on (0, 3).*
→ *Compute f(0) = 2 and f(3) = 27 - 9 - 18 + 2 = 2, so f(0) = f(3).*
→ *Find f'(x) = 3x squared minus 2x minus 6 and set it equal to zero.*
→ *Solve using the quadratic formula to get x = (1 + sqrt(19))/3 or (1 - sqrt(19))/3.*
→ *Only (1 + sqrt(19))/3 lies in (0, 3), so this is the value of c.*

Answer: $c = \frac{1 + \sqrt{19}}{3}$

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2. Verify Rolle's Theorem and find c for the function on the closed interval [-2, 2].

$$f(x) = x^2 - 4$$

→ *The function is a polynomial, so it is continuous and differentiable everywhere.*
→ *Compute f(-2) = 0 and f(2) = 0, so f(-2) = f(2).*
→ *Find f'(x) = 2x and set it equal to zero.*
→ *Solving gives x = 0, which lies in the open interval (-2, 2).*

Answer: $c = 0$

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3. Verify Rolle's Theorem and find c for the function on the closed interval [0, 2\pi].

$$f(x) = \sin(x)$$

→ *The sine function is continuous and differentiable on all real numbers.*
→ *Compute f(0) = 0 and f(2 pi) = 0, so the third condition is satisfied.*
→ *Find f'(x) = cos(x) and set it equal to zero.*
→ *Solve cos(x) = 0 on (0, 2 pi) to get x = pi/2 and 3 pi/2.*

Answer: $c = \frac{\pi}{2}, \frac{3\pi}{2}$

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4. Verify Rolle's Theorem and find c for the function on the closed interval [1, 3].

$$f(x) = x^2 - 4x + 3$$

→ *The function is a polynomial, so it is continuous on [1, 3] and differentiable on (1, 3).*
→ *Compute f(1) = 1 - 4 + 3 = 0 and f(3) = 9 - 12 + 3 = 0, so f(1) = f(3).*
→ *Find f'(x) = 2x - 4 and set it equal to zero.*
→ *Solving gives x = 2, which lies in (1, 3).*

Answer: $c = 2$

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5. Verify Rolle's Theorem and find c for the function on the closed interval [-1, 1].

$f(x) = x^3 - x$

→ *The function is a polynomial, so it is continuous and differentiable everywhere.*
→ *Compute f(-1) = -1 + 1 = 0 and f(1) = 1 - 1 = 0, so f(-1) = f(1).*
→ *Find f'(x) = 3x squared minus 1 and set it equal to zero.*
→ *Solve to get x squared = 1/3, so x = plus or minus sqrt(3)/3, both in (-1, 1).*

Answer: $c = \pm\frac{\sqrt{3}}{3}$

6. Verify Rolle's Theorem and find c for the function on the closed interval [0, 4].

$f(x) = x^2 - 4x + 1$

→ *The function is a polynomial, so it is continuous on [0, 4] and differentiable on (0, 4).*
→ *Compute f(0) = 1 and f(4) = 16 - 16 + 1 = 1, so f(0) = f(4).*
→ *Find f'(x) = 2x - 4 and set it equal to zero.*
→ *Solving gives x = 2, which lies in (0, 4).*

Answer: $c = 2$

7. Verify Rolle's Theorem and find c for the function on the closed interval [-3, 3].

$f(x) = 9 - x^2$

→ *The function is a polynomial, so it is continuous and differentiable everywhere.*
→ *Compute f(-3) = 0 and f(3) = 0, so f(-3) = f(3).*
→ *Find f'(x) = -2x and set it equal to zero.*
→ *Solving gives x = 0, which lies in (-3, 3).*

Answer: $c = 0$

8. Verify Rolle's Theorem and find c for the function on the closed interval [0, \pi].

$f(x) = \sin^2(x)$

→ *The function is continuous and differentiable on all real numbers.*
→ *Compute f(0) = 0 and f(pi) = 0, so f(0) = f(pi).*
→ *Find f'(x) = 2 sin(x) cos(x) = sin(2x) and set it equal to zero.*
→ *Solve sin(2x) = 0 on (0, pi) to get x = pi/2.*

Answer: $c = \frac{\pi}{2}$

9. Verify Rolle's Theorem and find c for the function on the closed interval [-2, 2].

$f(x) = x^4 - 2x^2$

→ *The function is a polynomial, so it is continuous and differentiable everywhere.*
→ *Compute f(-2) = 16 - 8 = 8 and f(2) = 16 - 8 = 8, so f(-2) = f(2).*
→ *Find f'(x) = 4x cubed minus 4x and factor as 4x(x squared minus 1).*
→ *Setting equal to zero gives x = 0, x = 1, and x = -1, all in (-2, 2).*

Answer: $c = 0, \pm 1$

10. Verify Rolle's Theorem and find c for the function on the closed interval [1, 5].

$f(x) = x^2 - 6x + 5$

→ *The function is a polynomial, so it is continuous on [1, 5] and differentiable on (1, 5).*
→ *Compute f(1) = 1 - 6 + 5 = 0 and f(5) = 25 - 30 + 5 = 0, so f(1) = f(5).*
→ *Find f'(x) = 2x - 6 and set it equal to zero.*
→ *Solving gives x = 3, which lies in (1, 5).*

Answer: $c = 3$