



# Second Derivatives Using Implicit Differentiation

Calculus Worksheet · Grade 11-12

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: / 10

## Learning Objectives

- Apply implicit differentiation to find the first derivative  $dy/dx$  of equations relating  $x$  and  $y$
- Use the quotient rule to differentiate rational expressions involving  $y$
- Simplify complex fractions to obtain a clean form of the second derivative

Find the first and second derivatives of each implicit equation, then simplify your final answer using algebra.

### 1. Find the second derivative of the following implicit equation.

$$x^2 + y^2 = 25$$

Answer: \_\_\_\_\_

### 2. Find the first derivative $dy/dx$ by implicit differentiation.

$$x^2 + y^2 = 49$$

Answer: \_\_\_\_\_

### 3. Find the second derivative of this circle equation.

$$x^2 + y^2 = 16$$

Answer: \_\_\_\_\_

### 4. Use implicit differentiation to find $dy/dx$ .

$$x^2 - y^2 = 9$$

Answer: \_\_\_\_\_

### 5. Find the second derivative for the given equation.

$$x^2 + 4y^2 = 36$$

Answer: \_\_\_\_\_

### 6. Find $dy/dx$ for the given relation.

$$xy = 12$$

Answer: \_\_\_\_\_

### 7. Find the second derivative of the relation.

$$xy = 10$$

Answer: \_\_\_\_\_



**8. Find  $dy/dx$  by implicit differentiation.**

$$x^3 + y^3 = 8$$

Answer: \_\_\_\_\_

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**9. Find the second derivative for the implicit equation.**

$$x^2 + y^2 = 1$$

Answer: \_\_\_\_\_

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**10. Find  $dy/dx$  for the equation below.**

$$x^2 + xy + y^2 = 7$$

Answer: \_\_\_\_\_





Remind students to substitute  $dy/dx$  back into the second derivative and to use the original equation to simplify when possible.

## Solutions

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1. Find the second derivative of the following implicit equation.

$$x^2 + y^2 = 25$$

- Differentiate both sides implicitly to get  $2x$  plus  $2y$  times  $dy/dx$  equals zero.
- Solve for  $dy/dx$  to obtain negative  $x$  over  $y$ .
- Apply the quotient rule to negative  $x$  over  $y$  to get the second derivative.
- Substitute  $dy/dx$  equals negative  $x$  over  $y$  and simplify using  $x$  squared plus  $y$  squared equals 25.

**Answer:** 
$$\frac{d^2y}{dx^2} = -\frac{25}{y^3}$$

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2. Find the first derivative  $dy/dx$  by implicit differentiation.

$$x^2 + y^2 = 49$$

- Differentiate each term with respect to  $x$ .
- The derivative of  $y$  squared is  $2y$  times  $dy/dx$ .
- Isolate  $dy/dx$  by dividing through.

**Answer:** 
$$\frac{dy}{dx} = -\frac{x}{y}$$

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3. Find the second derivative of this circle equation.

$$x^2 + y^2 = 16$$

- Find the first derivative using implicit differentiation to get negative  $x$  over  $y$ .
- Apply the quotient rule to differentiate negative  $x$  over  $y$ .
- Substitute the first derivative back in and combine fractions.
- Use the original equation to express the result in terms of  $y$  only.

**Answer:** 
$$\frac{d^2y}{dx^2} = -\frac{16}{y^3}$$

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4. Use implicit differentiation to find  $dy/dx$ .

$$x^2 - y^2 = 9$$

- Differentiate both sides with respect to  $x$ .
- The derivative of negative  $y$  squared is negative  $2y$  times  $dy/dx$ .
- Solve algebraically for  $dy/dx$ .

**Answer:** 
$$\frac{dy}{dx} = \frac{x}{y}$$

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5. Find the second derivative for the given equation.

$$x^2 + 4y^2 = 36$$

- Differentiate to find  $dy/dx$  equals negative  $x$  over four  $y$ .
- Apply the quotient rule to that expression.
- Substitute  $dy/dx$  back into the formula.
- Use the original equation to simplify the numerator.

**Answer:**  $\frac{d^2y}{dx^2} = -\frac{9}{4y^3}$

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6. Find  $dy/dx$  for the given relation.

$$xy = 12$$

- Differentiate using the product rule.
- You get  $y$  plus  $x$  times  $dy/dx$  equals zero.
- Isolate  $dy/dx$  by dividing.

**Answer:**  $\frac{dy}{dx} = -\frac{y}{x}$

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7. Find the second derivative of the relation.

$$xy = 10$$

- Differentiate implicitly to get  $dy/dx$  equals negative  $y$  over  $x$ .
- Apply the quotient rule to differentiate negative  $y$  over  $x$ .
- Substitute the first derivative into the resulting expression.
- Combine and simplify the fraction.

**Answer:**  $\frac{d^2y}{dx^2} = \frac{2y}{x^2}$

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8. Find  $dy/dx$  by implicit differentiation.

$$x^3 + y^3 = 8$$

- Differentiate both sides to get three  $x$  squared plus three  $y$  squared times  $dy/dx$  equals zero.
- Divide both sides by three.
- Solve for  $dy/dx$ .

**Answer:**  $\frac{dy}{dx} = -\frac{x^2}{y^2}$

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9. Find the second derivative for the implicit equation.

$$x^2 + y^2 = 1$$

- Differentiate to obtain  $dy/dx$  equals negative  $x$  over  $y$ .
- Apply the quotient rule to negative  $x$  over  $y$ .
- Substitute  $dy/dx$  back in and simplify the complex fraction.
- Use the identity  $x$  squared plus  $y$  squared equals one to simplify.

**Answer:**  $\frac{d^2y}{dx^2} = -\frac{1}{y^3}$

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10. Find  $dy/dx$  for the equation below.

$$x^2 + xy + y^2 = 7$$

→ Differentiate each term, using the product rule on  $xy$ .

→ Group all  $dy/dx$  terms on one side.

→ Factor and solve for  $dy/dx$ .

**Answer:**  $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$

