Tangent Lines to Curves Using Derivatives

Calculus Worksheet · Grade 11-12

Name: ____________________ Date: ______________ Score: / 10

Learning Objectives

- Apply derivative rules to trigonometric functions
- Evaluate derivatives at specific points to determine slope
- Write equations of tangent lines using point-slope form

Find the equation of the tangent line to each curve at the given point, expressing your answer in point-slope form.

1. Find the equation of the tangent line to the curve at the given point.

$y = \sec x$ at $\left(\frac{\pi}{3}, 2\right)$

Answer: ______________________________

2. Find the equation of the tangent line to the curve at the given point.

$y = \sec x - 2\cos x$ at $\left(\frac{\pi}{3}, 1\right)$

Answer: ______________________________

3. Find the equation of the tangent line to the curve at the given point.

$y = \sin x$ at $\left(\frac{\pi}{6}, \frac{1}{2}\right)$

Answer: ______________________________

4. Find the equation of the tangent line to the curve at the given point.

$y = \cos x$ at $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$

Answer: ______________________________

5. Find the equation of the tangent line to the curve at the given point.

$y = \tan x$ at $\left(\frac{\pi}{4}, 1\right)$

Answer: ______________________________

6. Find the equation of the tangent line to the curve at the given point.

$y = \csc x$ at $\left(\frac{\pi}{6}, 2\right)$

Answer: ______________________________

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7. Find the equation of the tangent line to the curve at the given point.

$y = x^2 + 3x$ at $(1, 4)$

Answer: ______________________________________

8. Find the equation of the tangent line to the curve at the given point.

$y = \sqrt{x}$ at $(4, 2)$

Answer: ______________________________________

9. Find the equation of the tangent line to the curve at the given point.

$y = \ln x$ at $(1, 0)$

Answer: ______________________________________

10. Find the equation of the tangent line to the curve at the given point.

$y = e^x$ at $(0, 1)$

Answer: ______________________________________

Students should be fluent with unit-circle values and derivative rules for trigonometric functions before attempting these problems.

Solutions

1. Find the equation of the tangent line to the curve at the given point.

$y = \sec x$ at $\left(\frac{\pi}{3}, 2\right)$

→ *Differentiate y with respect to x to get y prime equal to secant x times tangent x.*
→ *Evaluate the derivative at x equal to pi over three: secant of pi over three is 2 and tangent of pi over three is square root of three.*
→ *Multiply to find the slope m equals 2 times square root of three.*
→ *Substitute the point and slope into the point-slope form y minus y one equals m times x minus x one.*

Answer: $y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$

2. Find the equation of the tangent line to the curve at the given point.

$y = \sec x - 2\cos x$ at $\left(\frac{\pi}{3}, 1\right)$

→ *Differentiate term by term: derivative of secant x is secant x tangent x and derivative of negative two cosine x is two sine x.*
→ *Combine to get y prime equal to secant x tangent x plus two sine x.*
→ *Evaluate at pi over three: secant pi over three is 2, tangent pi over three is square root of three, and sine pi over three is square root of three over two.*
→ *Compute the slope: 2 times square root of three plus two times square root of three over two equals three square root of three.*
→ *Use point-slope form with the given point and slope to write the tangent line.*

Answer: $y - 1 = \left(2\sqrt{3} + \sqrt{3}\right)\left(x - \frac{\pi}{3}\right)$

3. Find the equation of the tangent line to the curve at the given point.

$y = \sin x$ at $\left(\frac{\pi}{6}, \frac{1}{2}\right)$

→ *Compute the derivative: y prime equals cosine x.*
→ *Evaluate cosine at pi over six to find the slope, which is square root of three over two.*
→ *Plug the slope and point into the point-slope formula.*

Answer: $y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

4. Find the equation of the tangent line to the curve at the given point.

$y = \cos x$ at $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$

→ *Differentiate to obtain y prime equal to negative sine x.*
→ *Evaluate at pi over four: negative sine of pi over four equals negative square root of two over two.*
→ *Substitute the slope and the point into the point-slope form.*

Answer: $y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

5. Find the equation of the tangent line to the curve at the given point.

$y = \tan x$ at $\left(\frac{\pi}{4}, 1\right)$

→ *Use the derivative rule for tangent: y prime equals secant squared x.*
→ *Evaluate at pi over four: secant of pi over four is square root of two, so secant squared is 2.*
→ *Apply the slope of 2 and the given point to the point-slope formula.*

Answer: $y - 1 = 2\left(x - \frac{\pi}{4}\right)$

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6. Find the equation of the tangent line to the curve at the given point.

$y = \csc x$ at $\left(\frac{\pi}{6}, 2\right)$

→ *Differentiate: y prime equals negative cosecant x times cotangent x.*
→ *Evaluate at pi over six: cosecant of pi over six is 2 and cotangent of pi over six is square root of three.*
→ *Multiply with the negative sign to get slope equal to negative two square root of three.*
→ *Write the tangent line using point-slope form.*

Answer: $y - 2 = -2\sqrt{3}\left(x - \frac{\pi}{6}\right)$

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7. Find the equation of the tangent line to the curve at the given point.

$y = x^2 + 3x$ at $(1, 4)$

→ *Apply the power rule to find y prime equal to two x plus three.*
→ *Evaluate at x equal to one to find the slope equal to five.*
→ *Insert the slope and the point into the point-slope formula.*

Answer: $y - 4 = 5(x - 1)$

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8. Find the equation of the tangent line to the curve at the given point.

$y = \sqrt{x}$ at $(4, 2)$

→ *Rewrite y as x to the one-half and differentiate to get y prime equal to one over two times the square root of x.*
→ *Evaluate at x equal to four: one over two times two equals one fourth.*
→ *Use the slope of one fourth with the given point in point-slope form.*

Answer: $y - 2 = \frac{1}{4}(x - 4)$

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9. Find the equation of the tangent line to the curve at the given point.

$y = \ln x$ at $(1, 0)$

→ *Recall the derivative of natural log of x is one over x.*
→ *Evaluate at x equal to one to find the slope equal to one.*
→ *Substitute into point-slope form to write the equation.*

Answer: $y - 0 = 1(x - 1)$

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10. Find the equation of the tangent line to the curve at the given point.

$y = e^x$ at $(0, 1)$

→ *The derivative of e to the x is e to the x itself.*
→ *Evaluate at x equal to zero: e to the zero equals one, so the slope is one.*
→ *Apply the slope and point to the point-slope form.*

Answer: $y - 1 = 1(x - 0)$