Antiderivatives of Exponential, Trigonometric, and Radical Functions

Calculus Worksheet · Grade 11-12

Name: ______________________ Date: ______________ Score: / 10

Learning Objectives

- Apply antiderivative rules for exponential and trigonometric functions
- Find antiderivatives of radical expressions by converting to fractional exponents
- Use initial conditions to determine the constant of integration

Find the antiderivative F(x) for each function below, remembering to include the constant of integration where appropriate.

1. Find the antiderivative of the function.

$$f'(x) = 3e^{x} + 7\sec^{2}(x)$$

Answer: ______________________

2. Find the antiderivative of the function.

$$f'(x) = \sqrt[4]{x^{3}} + \sqrt[3]{x^{4}}$$

Answer: ______________________

3. Find f(x) given f'(x) and the initial condition f(1) = 10.

$$f'(x) = \sqrt{x}(6x + 5)$$

Answer: ______________________

4. Find the antiderivative of the function.

$$f'(x) = 5\cos(x) - 2\sin(x)$$

Answer: ______________________

5. Find the antiderivative of the function.

$$f'(x) = \frac{4}{x} + 2e^{x}$$

Answer: ______________________

6. Find the antiderivative of the function.

$$f'(x) = \sqrt[3]{x^{2}} + \sqrt{x^{5}}$$

Answer: ______________________

7. Find the antiderivative of the function.

$$f'(x) = 8\sec(x)\tan(x) + 3\csc^{2}(x)$$

Answer: ______________________

8. Find f(x) given f'(x) and the initial condition f(0) = 5.

$f'(x) = 4e^{x} + 6x^{2}$

Answer: ____________________________________

9. Find the antiderivative of the function.

$f'(x) = \frac{7}{x} - 3\cos(x) + e^{x}$

Answer: ____________________________________

10. Find the antiderivative of the function.

$f'(x) = \sqrt{x}\left(x^{2} + \frac{1}{x}\right)$

Answer: ____________________________________

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ANSWER KEY & SOLUTIONS

Remind students to convert radicals to fractional exponents before applying the power rule and to always include +C unless an initial condition is given.

Solutions

1. Find the antiderivative of the function.

$f'(x) = 3e^x + 7\sec^2(x)$

→ *The antiderivative of e to the x is e to the x, so the first term becomes 3 e to the x.*
→ *The antiderivative of secant squared x is tangent x, so the second term becomes 7 tangent x.*
→ *Combine the results and add the constant of integration C.*

Answer: $f(x) = 3e^x + 7\tan(x) + C$

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2. Find the antiderivative of the function.

$f'(x) = \sqrt[4]{x^3} + \sqrt[3]{x^4}$

→ *Rewrite each radical with fractional exponents: x to the 3/4 power and x to the 4/3 power.*
→ *Apply the power rule by adding 1 to each exponent, giving 7/4 and 7/3.*
→ *Divide each term by its new exponent, producing 4/7 and 3/7 as the leading coefficients.*
→ *Convert the fractional exponents back to radical form and add the constant C.*

Answer: $f(x) = \frac{4}{7}\sqrt[4]{x^7} + \frac{3}{7}\sqrt[3]{x^7} + C$

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3. Find f(x) given f'(x) and the initial condition f(1) = 10.

$f'(x) = \sqrt{x}(6x + 5)$

→ *Rewrite the square root of x as x to the 1/2 power and distribute to get 6 x to the 3/2 plus 5 x to the 1/2.*
→ *Apply the power rule to each term, adding 1 to the exponents and dividing by the new exponents.*
→ *This produces 12/5 x to the 5/2 plus 10/3 x to the 3/2 plus C.*
→ *Substitute x = 1 and set the expression equal to 10 to solve for C, giving C = 4/15.*

Answer: $f(x) = \frac{12}{5}x^{5/2} + \frac{10}{3}x^{3/2} + \frac{4}{15}$

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4. Find the antiderivative of the function.

$f'(x) = 5\cos(x) - 2\sin(x)$

→ *The antiderivative of cosine x is sine x, so the first term becomes 5 sine x.*
→ *The antiderivative of sine x is negative cosine x, so subtracting gives positive 2 cosine x.*
→ *Add the constant of integration C to complete the antiderivative.*

Answer: $f(x) = 5\sin(x) + 2\cos(x) + C$

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5. Find the antiderivative of the function.

$$f'(x) = \frac{4}{x} + 2e^x$$

→ *The antiderivative of 1 over x is the natural log of the absolute value of x, so the first term becomes 4 times ln of absolute value of x.*
→ *The antiderivative of e to the x is e to the x, so the second term becomes 2 e to the x.*
→ *Add the constant of integration C.*

Answer: $f(x) = 4\ln|x| + 2e^x + C$

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6. Find the antiderivative of the function.

$$f'(x) = \sqrt[3]{x^2} + \sqrt{x^5}$$

→ *Rewrite each radical with fractional exponents: x to the 2/3 power and x to the 5/2 power.*
→ *Apply the power rule by adding 1 to each exponent, giving 5/3 and 7/2.*
→ *Divide each term by its new exponent, producing coefficients 3/5 and 2/7.*
→ *Rewrite the fractional exponents back as radicals and add the constant C.*

Answer: $f(x) = \frac{3}{5}\sqrt[3]{x^5} + \frac{2}{7}\sqrt{x^7} + C$

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7. Find the antiderivative of the function.

$$f'(x) = 8\sec(x)\tan(x) + 3\csc^2(x)$$

→ *The antiderivative of secant x tangent x is secant x, so the first term becomes 8 secant x.*
→ *The antiderivative of cosecant squared x is negative cotangent x, so the second term becomes negative 3 cotangent x.*
→ *Add the constant of integration C.*

Answer: $f(x) = 8\sec(x) - 3\cot(x) + C$

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8. Find f(x) given f'(x) and the initial condition f(0) = 5.

$$f'(x) = 4e^x + 6x^2$$

→ *The antiderivative of 4 e to the x is 4 e to the x.*
→ *Apply the power rule to 6 x squared, adding 1 to the exponent and dividing by 3 to obtain 2 x cubed.*
→ *Substitute x = 0 and set the expression equal to 5, giving 4 plus C equals 5 so C equals 1.*

Answer: $f(x) = 4e^x + 2x^3 + 1$

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9. Find the antiderivative of the function.

$$f'(x) = \frac{7}{x} - 3\cos(x) + e^x$$

→ *The antiderivative of 7 over x is 7 times the natural log of the absolute value of x.*
→ *The antiderivative of cosine x is sine x, so subtracting gives negative 3 sine x.*
→ *The antiderivative of e to the x is e to the x.*
→ *Add the constant of integration C.*

Answer: $f(x) = 7\ln|x| - 3\sin(x) + e^x + C$

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10. Find the antiderivative of the function.

$f'(x) = \sqrt{x}\left(x^2 + \frac{1}{x}\right)$

→ *Rewrite the square root of x as x to the 1/2 and distribute to get x to the 5/2 plus x to the negative 1/2.*
→ *Apply the power rule to x to the 5/2 by adding 1 and dividing, producing 2/7 x to the 7/2.*
→ *Apply the power rule to x to the negative 1/2 by adding 1 and dividing, producing 2 x to the 1/2 which is 2 square root of x.*
→ *Add the constant of integration C.*

Answer: $f(x) = \frac{2}{7}x^{7/2} + 2\sqrt{x} + C$