Areas Between Curves Using Sub-Regions

Calculus Worksheet · Grade 11-12

Name: ______________________________ Date: _______________ Score: / 10

Learning Objectives

- Identify sub-regions formed by intersecting curves and lines
- Set up definite integrals using the top-minus-bottom function method
- Evaluate definite integrals to compute total area between curves

For each problem, sketch the region, identify sub-regions if needed, set up the appropriate definite integral(s), and evaluate to find the area.

1. Find the area of the region in the first quadrant bounded above by y = √x, below by the x-axis, and by the line y = x - 2.

$$A = \int_0^2 \sqrt{x}\,dx + \int_2^4 (\sqrt{x} - (x-2))dx$$

Answer: ________________________________

2. Find the area of the region bounded by $y = x^2$ and y = x between x = 0 and x = 1.

$$A = \int_0^1 (x - x^2)dx$$

Answer: ________________________________

3. Find the area of the region bounded above by $y = 4 - x^2$ and below by the x-axis.

$$A = \int_{-2}^2 (4 - x^2)dx$$

Answer: ________________________________

4. Find the area of the region bounded by y = √x, the y-axis, and the line y = 2.

$$A = \int_0^2 y^2\,dy$$

Answer: ________________________________

5. Find the area of the region bounded by $y = x^3$ and y = x in the first quadrant.

$$A = \int_0^1 (x - x^3)dx$$

Answer: ________________________________

6. Find the area of the region bounded by y = x + 2, y = x², and the y-axis using sub-regions if needed.

$$A = \int_0^2 \left((x+2) - x^2\right) dx$$

Answer: ________________________________

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7. Find the area of the region bounded by y = √x, y = x - 2, and the line x = 0 using sub-regions.

$$A = \int_0^2 \sqrt{x}\, dx + \int_2^4 (\sqrt{x} - (x-2)) dx$$

Answer: ________________________________

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8. Find the area enclosed between y = sin(x) and the x-axis from x = 0 to x = π.

$$A = \int_0^\pi \sin(x)\, dx$$

Answer: ________________________________

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9. Find the area of the region bounded by y = x² and y = 2x using top-minus-bottom.

$$A = \int_0^2 (2x - x^2) dx$$

Answer: ________________________________

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10. Find the area of the region bounded by y = √x, y = 6 - x, and the x-axis using sub-regions.

$$A = \int_0^4 \sqrt{x}\, dx + \int_4^6 (6 - x)\, dx$$

Answer: ________________________________

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ANSWER KEY & SOLUTIONS

Emphasize that students must sketch graphs first and check whether the bottom (or top) function changes across the interval — this determines whether sub-regions are required.

Solutions

1. Find the area of the region in the first quadrant bounded above by y = √x, below by the x-axis, and by the line y = x - 2.

$$A = \int_0^2 \sqrt{x}\,dx + \int_2^4 (\sqrt{x} - (x-2))dx$$

→ *Sketch y = √x, y = x - 2, and the x-axis to visualize two sub-regions.*
→ *Find the intersection of y = √x and y = x - 2 by solving √x = x - 2, giving x = 4.*
→ *For the first sub-region from x = 0 to x = 2, integrate √x with the x-axis as the bottom.*
→ *For the second sub-region from x = 2 to x = 4, integrate √x minus (x - 2).*
→ *Evaluate each integral and add the results to get the total area of 10/3.*

Answer: $\frac{10}{3}$

2. Find the area of the region bounded by y = x^2 and y = x between x = 0 and x = 1.

$$A = \int_0^1 (x - x^2)dx$$

→ *Sketch both curves and note y = x is above y = x^2 on [0,1].*
→ *Set up the integral of top minus bottom: x - x^2.*
→ *Integrate to obtain x^2/2 - x^3/3.*
→ *Evaluate from 0 to 1 to get 1/2 - 1/3 = 1/6.*

Answer: $\frac{1}{6}$

3. Find the area of the region bounded above by y = 4 - x^2 and below by the x-axis.

$$A = \int_{-2}^2 (4 - x^2)dx$$

→ *Find where y = 4 - x^2 meets the x-axis by solving 4 - x^2 = 0, giving x = ±2.*
→ *Set up the integral of 4 - x^2 from -2 to 2.*
→ *Integrate to get 4x - x^3/3.*
→ *Evaluate at the bounds: (8 - 8/3) - (-8 + 8/3) = 32/3.*

Answer: $\frac{32}{3}$

4. Find the area of the region bounded by y = √x, the y-axis, and the line y = 2.

$$A = \int_0^2 y^2\,dy$$

→ *Sketch the region and note it is easier to integrate with respect to y.*
→ *Rewrite y = √x as x = y^2, which is the right boundary.*
→ *Set up the integral of y^2 from y = 0 to y = 2.*
→ *Integrate to get y^3/3 and evaluate to obtain 8/3.*

Answer: $\frac{8}{3}$

5. Find the area of the region bounded by $y = x^3$ and $y = x$ in the first quadrant.

$$A = \int_0^1 (x - x^3)dx$$

→ *Find intersections by solving $x = x^3$, giving $x = 0$ and $x = 1$ in the first quadrant.*
→ *Identify $y = x$ as the top function and $y = x^3$ as the bottom on [0,1].*
→ *Integrate $x - x^3$ to get $x^2/2 - x$■$/4$.*
→ *Evaluate from 0 to 1 to get 1/2 - 1/4 = 1/4.*

Answer: $\frac{1}{4}$

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6. Find the area of the region bounded by $y = x + 2$, $y = x^2$, and the y-axis using sub-regions if needed.

$$A = \int_0^2 ((x + 2) - x^2)dx$$

→ *Find the intersection of $y = x + 2$ and $y = x^2$ by solving $x^2 = x + 2$, giving $x = 2$ (in first quadrant).*
→ *Determine that $y = x + 2$ is above $y = x^2$ on [0,2].*
→ *Set up the integral of $(x + 2) - x^2$ from 0 to 2.*
→ *Integrate to get $x^2/2 + 2x - x^3/3$ and evaluate to obtain 10/3.*

Answer: $\frac{10}{3}$

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7. Find the area of the region bounded by $y = \sqrt{x}$, $y = x - 2$, and the line $x = 0$ using sub-regions.

$$A = \int_0^2 \sqrt{x}\,dx + \int_2^4 (\sqrt{x} - (x - 2))dx$$

→ *Sketch the curves to see two sub-regions are required.*
→ *From $x = 0$ to $x = 2$, the bottom is the x-axis since $y = x - 2$ is below it.*
→ *From $x = 2$ to $x = 4$, the bottom changes to $y = x - 2$.*
→ *Compute each integral separately and add to get 10/3.*

Answer: $\frac{10}{3}$

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8. Find the area enclosed between $y = \sin(x)$ and the x-axis from $x = 0$ to $x = \pi$.

$$A = \int_0^{\pi} \sin(x)\,dx$$

→ *Note that sin(x) is non-negative on [0,π], so no sub-regions are needed.*
→ *Set up the definite integral of sin(x) from 0 to π.*
→ *The antiderivative of sin(x) is -cos(x).*
→ *Evaluate: $-\cos(\pi) - (-\cos(0)) = 1 + 1 = 2$.*

Answer: 2

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9. Find the area of the region bounded by $y = x^2$ and $y = 2x$ using top-minus-bottom.

$$A = \int_0^2 (2x - x^2)dx$$

→ *Find intersections by solving $x^2 = 2x$, giving $x = 0$ and $x = 2$.*
→ *Determine $y = 2x$ is above $y = x^2$ on [0,2].*
→ *Set up the integral of $2x - x^2$ from 0 to 2.*
→ *Integrate to get $x^2 - x^3/3$ and evaluate to obtain 4 - 8/3 = 4/3.*

Answer: $\frac{4}{3}$

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10. Find the area of the region bounded by $y = \sqrt{x}$, $y = 6 - x$, and the x-axis using sub-regions.

$$A = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6 - x)\,dx$$

→ *Find the intersection of $y = \sqrt{x}$ and $y = 6 - x$ by solving $\sqrt{x} = 6 - x$, giving $x = 4$.*
→ *From $x = 0$ to $x = 4$, the top is $y = \sqrt{x}$ and the bottom is the x-axis.*
→ *From $x = 4$ to $x = 6$, the top is $y = 6 - x$ and the bottom is the x-axis.*
→ *Evaluate both integrals: $16/3 + 2 = 22/3$.*

Answer: $\frac{22}{3}$