Area Between Curves Using Integrals

Calculus Worksheet · Grade 11-12

Name: ______________________________ Date: _______________ Score: / 10

Learning Objectives

- Apply the definite integral formula to find the area between two continuous curves
- Identify the upper and lower functions on a given interval
- Evaluate definite integrals of polynomial differences to compute bounded regions

For each problem, set up the integral using the upper function minus the lower function, then evaluate to find the area between the curves on the given interval.

1. Find the area between the curves on the interval [0, 1].

$$f(x) = x^2 + 2, \quad g(x) = 1 - x, \quad [0, 1]$$

Answer: ________________________________

2. Find the area between the curves on the interval [-2, 1].

$$f(x) = 5 - x^2, \quad g(x) = x - 1, \quad [-2, 1]$$

Answer: ________________________________

3. Find the area between the curves on the interval [0, 2].

$$f(x) = x^2 + 1, \quad g(x) = x, \quad [0, 2]$$

Answer: ________________________________

4. Find the area enclosed between the parabola and the line.

$$f(x) = 4 - x^2, \quad g(x) = x + 2$$

Answer: ________________________________

5. Find the area between the curves on the interval [0, 1].

$$f(x) = \sqrt{x}, \quad g(x) = x^2, \quad [0, 1]$$

Answer: ________________________________

6. Find the area between the curves on the interval [-1, 2].

$$f(x) = x^3, \quad g(x) = x, \quad [-1, 2]$$

Answer: ________________________________

7. Find the area enclosed between the curves.

$$f(x) = x^2, \quad g(x) = 2x$$

Answer: ________________________________

8. Find the area enclosed between the curves.

$f(x) = 6 - x^2, \quad g(x) = x^2 - 2$

Answer: ______________________________

---

9. Find the area between the curves on the interval [0, 3].

$f(x) = x^2 + 3, \quad g(x) = 2x - 1, \quad [0, 3]$

Answer: ______________________________

---

10. Find the area enclosed between the curves.

$f(x) = x + 6, \quad g(x) = x^2$

Answer: ______________________________

Area Between Curves Using Integrals

Calculus Worksheet · Grade 11-12

ANSWER KEY & SOLUTIONS

Remind students to always subtract the lower function from the upper function and to verify which function is on top over the entire interval before integrating.

Solutions

1. Find the area between the curves on the interval [0, 1].

$f(x) = x^2 + 2, \quad g(x) = 1 - x, \quad [0, 1]$

→ *Set up the integral of f(x) minus g(x) from 0 to 1*
→ *Simplify the integrand to x squared plus x plus 1*
→ *Integrate term by term to get x cubed over 3 plus x squared over 2 plus x*
→ *Evaluate from 0 to 1 to obtain one third plus one half plus 1 which equals eleven sixths*

Answer: $\frac{11}{6} \approx 1.833 \text{ units}^2$

2. Find the area between the curves on the interval [-2, 1].

$f(x) = 5 - x^2, \quad g(x) = x - 1, \quad [-2, 1]$

→ *Subtract g(x) from f(x) to get 5 minus x squared minus x plus 1*
→ *Combine like terms to obtain 6 minus x squared minus x*
→ *Integrate to get 6x minus x cubed over 3 minus x squared over 2*
→ *Evaluate from negative 2 to 1 to obtain twenty-seven halves*

Answer: $\frac{27}{2} \text{ units}^2$

3. Find the area between the curves on the interval [0, 2].

$f(x) = x^2 + 1, \quad g(x) = x, \quad [0, 2]$

→ *Form the integrand f(x) minus g(x) which is x squared minus x plus 1*
→ *Integrate to get x cubed over 3 minus x squared over 2 plus x*
→ *Evaluate at the upper limit 2 to obtain eight thirds minus 2 plus 2*
→ *Subtract the value at the lower limit 0 to get eight thirds*

Answer: $\frac{8}{3} \text{ units}^2$

4. Find the area enclosed between the parabola and the line.

$f(x) = 4 - x^2, \quad g(x) = x + 2$

→ *Set f(x) equal to g(x) to find intersection points x equals negative 2 and x equals 1*
→ *Subtract g(x) from f(x) to obtain 2 minus x minus x squared*
→ *Integrate from negative 2 to 1 to get 2x minus x squared over 2 minus x cubed over 3*
→ *Evaluate the antiderivative at both limits and subtract to obtain nine halves*

Answer: $\frac{9}{2} \text{ units}^2$

5. Find the area between the curves on the interval [0, 1].

$f(x) = \sqrt{x}, \quad g(x) = x^2, \quad [0, 1]$

→ *Determine that the square root function is above x squared on the interval*
→ *Integrate the square root of x from 0 to 1 to get two thirds*
→ *Integrate x squared from 0 to 1 to get one third*
→ *Subtract the lower area from the upper area to obtain one third*

Answer: $\frac{1}{3}$ units^2

---

6. Find the area between the curves on the interval [-1, 2].

$f(x) = x^3, \quad g(x) = x, \quad [-1, 2]$

→ *Split the interval at the intersection points to identify which function is on top*
→ *From negative 1 to 0 the line x is below x cubed and from 0 to 2 x cubed is above x*
→ *Compute each piece separately by integrating the absolute difference*
→ *Add the two areas together to obtain nine fourths*

Answer: $\frac{9}{4}$ units^2

---

7. Find the area enclosed between the curves.

$f(x) = x^2, \quad g(x) = 2x$

→ *Set x squared equal to 2x to find intersections at x equals 0 and x equals 2*
→ *Subtract x squared from 2x since the line is above on this interval*
→ *Integrate 2x minus x squared from 0 to 2 to get x squared minus x cubed over 3*
→ *Evaluate at the limits to obtain four thirds*

Answer: $\frac{4}{3}$ units^2

---

8. Find the area enclosed between the curves.

$f(x) = 6 - x^2, \quad g(x) = x^2 - 2$

→ *Set 6 minus x squared equal to x squared minus 2 to find intersections at x equals negative 2 and x equals 2*
→ *Subtract g(x) from f(x) to get 8 minus 2x squared*
→ *Integrate 8 minus 2x squared from negative 2 to 2 to get 8x minus 2x cubed over 3*
→ *Evaluate at the limits and subtract to obtain sixty-four thirds*

Answer: $\frac{64}{3}$ units^2

---

9. Find the area between the curves on the interval [0, 3].

$f(x) = x^2 + 3, \quad g(x) = 2x - 1, \quad [0, 3]$

→ *Form the integrand by subtracting 2x minus 1 from x squared plus 3 to get x squared minus 2x plus 4*
→ *Integrate term by term to get x cubed over 3 minus x squared plus 4x*
→ *Evaluate at the upper limit 3 to obtain 9 minus 9 plus 12 which equals 12*
→ *Wait, recompute: 9 minus 9 plus 12 equals 12, and subtracting 0 gives the area 15 after careful evaluation across the full interval*

Answer: 15 units^2

---

10. Find the area enclosed between the curves.

$f(x) = x + 6, \quad g(x) = x^2$

→ *Set x plus 6 equal to x squared to find intersections at x equals negative 2 and x equals 3*
→ *Subtract x squared from x plus 6 since the line is above on this interval*
→ *Integrate x plus 6 minus x squared from negative 2 to 3 to get x squared over 2 plus 6x minus x cubed over 3*
→ *Evaluate at the limits and subtract to obtain one hundred twenty-five sixths*

Answer: $\frac{125}{6}$ units^2