Area Under the Curve of Absolute Value Functions

Calculus Worksheet · Grade 11-12

Name: ______________________________ Date: _______________ Score: / 9

Learning Objectives

- Find x-intercepts of absolute value functions to identify where the graph changes sign
- Split definite integrals of absolute value functions into piecewise integrals over appropriate subintervals
- Evaluate definite integrals to compute the exact area under absolute value curves

For each problem, find the x-intercept of the absolute value function, split the integral at that point, and evaluate the resulting definite integrals to find the total area under the curve.

1. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{0}^{2} |2x - 1|\, dx$$

Answer: _________________________________

2. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{0}^{3} |x - 1|\, dx$$

Answer: _________________________________

3. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{-2}^{2} |x|\, dx$$

Answer: _________________________________

4. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{0}^{4} |3x - 6|\, dx$$

Answer: _________________________________

5. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{-1}^{3} |x - 2|\, dx$$

Answer: _________________________________

6. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{-3}^{3} |2x|\, dx$$

Answer: _________________________________

7. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{0}^{5} |x - 3|\, dx$$

Answer: ______________________________________

8. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{-2}^{4} |x + 1|\, dx$$

Answer: ______________________________________

9. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{1}^{5} |2x - 4|\, dx$$

Answer: ______________________________________

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ANSWER KEY & SOLUTIONS

Remind students that |f(x)| equals -f(x) where f(x) < 0 and f(x) where f(x) >= 0, so the integral must be split at every zero of f(x) within the interval.

Solutions

1. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_0^2 |2x - 1|\, dx$$

- → *Find the x-intercept by setting 2x minus 1 equal to zero, giving x equals one-half*
- → *Split the integral at x equals one-half: from 0 to one-half use negative of 2x minus 1, and from one-half to 2 use 2x minus 1*
- → *Integrate the first piece to get negative x squared plus x evaluated from 0 to one-half, which equals one-fourth*
- → *Integrate the second piece to get x squared minus x evaluated from one-half to 2, which equals nine-fourths*
- → *Add the two areas: one-fourth plus nine-fourths equals five-halves*

Answer: $\frac{5}{2}$

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2. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_0^3 |x - 1|\, dx$$

- → *Set x minus 1 equal to zero to find the x-intercept at x equals 1*
- → *Split the integral at x equals 1: from 0 to 1 use negative of x minus 1, and from 1 to 3 use x minus 1*
- → *Integrate the first piece to get negative one-half x squared plus x evaluated from 0 to 1, which equals one-half*
- → *Integrate the second piece to get one-half x squared minus x evaluated from 1 to 3, which equals 2*
- → *Add the two areas: one-half plus 2 equals five-halves*

Answer: $\frac{5}{2}$

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3. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{-2}^2 |x|\, dx$$

- → *The x-intercept is at x equals 0, which lies inside the interval*
- → *Split the integral at x equals 0: from negative 2 to 0 use negative x, and from 0 to 2 use x*
- → *Integrate the first piece to get negative one-half x squared evaluated from negative 2 to 0, which equals 2*
- → *Integrate the second piece to get one-half x squared evaluated from 0 to 2, which equals 2*
- → *Add the two areas: 2 plus 2 equals 4*

Answer: 4

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4. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_0^4 |3x - 6|\, dx$$

→ *Set 3x minus 6 equal to zero to find the x-intercept at x equals 2*
→ *Split the integral at x equals 2: from 0 to 2 use negative of 3x minus 6, and from 2 to 4 use 3x minus 6*
→ *Integrate the first piece to get negative three-halves x squared plus 6x evaluated from 0 to 2, which equals 6*
→ *Integrate the second piece to get three-halves x squared minus 6x evaluated from 2 to 4, which equals 6*
→ *Add the two areas: 6 plus 6 equals 12*

Answer: 12

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5. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{-1}^3 |x - 2|\, dx$$

→ *Set x minus 2 equal to zero to find the x-intercept at x equals 2*
→ *Split the integral at x equals 2: from negative 1 to 2 use negative of x minus 2, and from 2 to 3 use x minus 2*
→ *Integrate the first piece to get negative one-half x squared plus 2x evaluated from negative 1 to 2, which equals nine-halves*
→ *Integrate the second piece to get one-half x squared minus 2x evaluated from 2 to 3, which equals one-half*
→ *Add the two areas: nine-halves plus one-half equals 5*

Answer: 5

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6. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_{-3}^3 |2x|\, dx$$

→ *The x-intercept is at x equals 0, which lies inside the interval*
→ *Split the integral at x equals 0: from negative 3 to 0 use negative 2x, and from 0 to 3 use 2x*
→ *Integrate the first piece to get negative x squared evaluated from negative 3 to 0, which equals 9*
→ *Integrate the second piece to get x squared evaluated from 0 to 3, which equals 9*
→ *Add the two areas: 9 plus 9 equals 18*

Answer: 18

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7. Evaluate the definite integral of the absolute value function over the given interval.

$$\int_0^5 |x - 3|\, dx$$

→ *Set x minus 3 equal to zero to find the x-intercept at x equals 3*
→ *Split the integral at x equals 3: from 0 to 3 use negative of x minus 3, and from 3 to 5 use x minus 3*
→ *Integrate the first piece to get negative one-half x squared plus 3x evaluated from 0 to 3, which equals nine-halves*
→ *Integrate the second piece to get one-half x squared minus 3x evaluated from 3 to 5, which equals 2*
→ *Add the two areas: nine-halves plus 2 equals thirteen-halves*

Answer: $\frac{13}{2}$

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8. Evaluate the definite integral of the absolute value function over the given interval.

$\int_{-2}^{4} |x + 1| \, dx$

→ *Set x plus 1 equal to zero to find the x-intercept at x equals negative 1*

→ *Split the integral at x equals negative 1: from negative 2 to negative 1 use negative of x plus 1, and from negative 1 to 4 use x plus 1*

→ *Integrate the first piece to get negative one-half x squared minus x evaluated from negative 2 to negative 1, which equals one-half*

→ *Integrate the second piece to get one-half x squared plus x evaluated from negative 1 to 4, which equals fourteen*

→ *Add the two areas: one-half plus fourteen equals twenty-nine-halves*

Answer: $\frac{29}{2}$

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9. Evaluate the definite integral of the absolute value function over the given interval.

$\int_{1}^{5} |2x - 4| \, dx$

→ *Set 2x minus 4 equal to zero to find the x-intercept at x equals 2*

→ *Split the integral at x equals 2: from 1 to 2 use negative of 2x minus 4, and from 2 to 5 use 2x minus 4*

→ *Integrate the first piece to get negative x squared plus 4x evaluated from 1 to 2, which equals 1*

→ *Integrate the second piece to get x squared minus 4x evaluated from 2 to 5, which equals 9*

→ *Add the two areas: 1 plus 9 equals 10*

Answer: 10