Average Value of a Function

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Apply the average value formula for a continuous function on a closed interval
- Evaluate definite integrals to compute the average value of polynomial and transcendental functions
- Use the Mean Value Theorem for Integrals to find a value c where f(c) equals the average value

For each problem, use the average value formula f_avg = 1/(b-a) integral from a to b of f(x) dx, show all integration steps, and simplify your final answer.

1. Find the average value of the function on the interval [1, 4].

$f(x) = 3x^2 - 2x, \quad [1, 4]$

Answer: _____

2. Find the value of c in [0, 2] such that f(c) equals the average value of the function.

$f(x) = 4 - x^2, \quad [0, 2]$

Answer: _____

3. Find the average value of the function on the interval [0, 3].

$f(x) = x^2 + 1, \quad [0, 3]$

Answer: _____

4. Find the average value of the linear function on the interval [2, 6].

$f(x) = 2x + 1, \quad [2, 6]$

Answer: _____

5. Find the average value of the trigonometric function on the interval [0, pi].

$f(x) = \sin(x), \quad [0, \pi]$

Answer: _____

6. Find the average value of the rational function on the interval [1, e].

$f(x) = \frac{1}{x}, \quad [1, e]$

Answer: _____

7. Find the value of c in [1, 3] such that f(c) equals the average value of the function.

$f(x) = x^2, \quad [1, 3]$

Answer: _____

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8. Find the average value of the exponential function on the interval [0, 2].

$f(x) = e^x, \quad [0, 2]$

Answer: _____

9. Find the average value of the cubic function on the interval [-1, 2].

$f(x) = x^3 - x, \quad [-1, 2]$

Answer: _____

10. Find the average value of the square root function on the interval [0, 4].

$f(x) = \sqrt{x}, \quad [0, 4]$

Answer: _____

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ANSWER KEY & SOLUTIONS

Emphasize the connection between the average value formula and the Mean Value Theorem for Integrals; remind students to check that c lies inside the given interval.

Solutions

1. Find the average value of the function on the interval [1, 4].

$f(x) = 3x^2 - 2x, \quad [1, 4]$

→ *Write the average value formula: 1 divided by (b minus a) times the integral from a to b of f of x dx.*
→ *Substitute a equals 1, b equals 4, giving one-third times the integral from 1 to 4 of (3x squared minus 2x) dx.*
→ *Integrate to get x cubed minus x squared evaluated from 1 to 4.*
→ *Evaluate the antiderivative: (64 minus 16) minus (1 minus 1) equals 48.*
→ *Multiply by one-third to obtain the average value of 16.*

Answer: $f_{avg} = 16$

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2. Find the value of c in [0, 2] such that f(c) equals the average value of the function.

$f(x) = 4 - x^2, \quad [0, 2]$

→ *Compute the average value using one-half times the integral from 0 to 2 of (4 minus x squared) dx.*
→ *Integrate to get 4x minus x cubed over 3, evaluated from 0 to 2, which equals 16 over 3.*
→ *Multiply by one-half to find the average value equals 8 over 3.*
→ *Set f of c equal to 8 over 3, so 4 minus c squared equals 8 over 3.*
→ *Solve to get c squared equals 4 over 3, so c equals 2 over the square root of 3, which is approximately 1.155 and lies in [0, 2].*

Answer: $c = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

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3. Find the average value of the function on the interval [0, 3].

$f(x) = x^2 + 1, \quad [0, 3]$

→ *Apply the formula: one-third times the integral from 0 to 3 of (x squared plus 1) dx.*
→ *Integrate to obtain x cubed over 3 plus x, evaluated from 0 to 3.*
→ *Evaluate to get 9 plus 3, which equals 12.*
→ *Multiply by one-third to find the average value equals 4.*

Answer: $f_{avg} = 4$

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4. Find the average value of the linear function on the interval [2, 6].

$f(x) = 2x + 1, \quad [2, 6]$

→ *Apply the formula: one-fourth times the integral from 2 to 6 of (2x plus 1) dx.*
→ *Integrate to obtain x squared plus x, evaluated from 2 to 6.*
→ *Evaluate: (36 plus 6) minus (4 plus 2) equals 36.*
→ *Multiply by one-fourth to find the average value equals 9.*

Answer: $f_{avg} = 9$

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5. Find the average value of the trigonometric function on the interval [0, pi].

$f(x) = \sin(x), \quad [0, \pi]$

→ *Apply the formula: 1 over pi times the integral from 0 to pi of sine x dx.*
→ *Integrate sine x to get negative cosine x, evaluated from 0 to pi.*
→ *Evaluate: negative cosine of pi minus negative cosine of 0 equals 1 plus 1, which equals 2.*
→ *Multiply by 1 over pi to find the average value equals 2 over pi.*

Answer: $f_{avg} = \frac{2}{\pi}$

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6. Find the average value of the rational function on the interval [1, e].

$f(x) = \frac{1}{x}, \quad [1, e]$

→ *Apply the formula: 1 over (e minus 1) times the integral from 1 to e of one over x dx.*
→ *Integrate to obtain the natural logarithm of x, evaluated from 1 to e.*
→ *Evaluate: ln of e minus ln of 1 equals 1 minus 0, which equals 1.*
→ *Multiply by 1 over (e minus 1) to find the average value equals 1 over (e minus 1).*

Answer: $f_{avg} = \frac{1}{e-1}$

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7. Find the value of c in [1, 3] such that f(c) equals the average value of the function.

$f(x) = x^2, \quad [1, 3]$

→ *Compute the average value: one-half times the integral from 1 to 3 of x squared dx.*
→ *Integrate to obtain x cubed over 3, evaluated from 1 to 3, which equals 27 over 3 minus 1 over 3, or 26 over 3.*
→ *Multiply by one-half to get the average value equals 13 over 3.*
→ *Set c squared equal to 13 over 3 and solve to get c equals the square root of 13 over 3, approximately 2.08, which lies in [1, 3].*

Answer: $c = \sqrt{\frac{13}{3}}$

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8. Find the average value of the exponential function on the interval [0, 2].

$f(x) = e^x, \quad [0, 2]$

→ *Apply the formula: one-half times the integral from 0 to 2 of e to the x dx.*
→ *Integrate to obtain e to the x, evaluated from 0 to 2.*
→ *Evaluate: e squared minus 1.*
→ *Multiply by one-half to find the average value equals (e squared minus 1) over 2.*

Answer: $f_{avg} = \frac{e^2 - 1}{2}$

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9. Find the average value of the cubic function on the interval [-1, 2].

$f(x) = x^3 - x, \quad [-1, 2]$

→ *Apply the formula: one-third times the integral from negative 1 to 2 of (x cubed minus x) dx.*
→ *Integrate to obtain x to the fourth over 4 minus x squared over 2, evaluated from negative 1 to 2.*
→ *Evaluate the upper limit: 16 over 4 minus 4 over 2 equals 4 minus 2, or 2.*
→ *Evaluate the lower limit: 1 over 4 minus 1 over 2 equals negative 1 over 4.*
→ *Subtract: 2 minus negative one-fourth equals 9 over 4, and multiply by one-third to obtain three-fourths.*

Answer: $f_{avg} = \frac{3}{4}$

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10. Find the average value of the square root function on the interval [0, 4].

$f(x) = \sqrt{x}, \quad [0, 4]$

→ *Apply the formula: one-fourth times the integral from 0 to 4 of the square root of x dx.*
→ *Rewrite the square root of x as x to the one-half and integrate to obtain two-thirds times x to the three-halves.*
→ *Evaluate from 0 to 4: two-thirds times 8 equals 16 over 3.*
→ *Multiply by one-fourth to find the average value equals 4 over 3.*

Answer: $f_{avg} = \frac{4}{3}$