Evaluating Differential Equations

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Find general solutions to first-order differential equations by integration
- Determine particular solutions using initial conditions
- Apply integration techniques to solve separable differential equations

Solve each differential equation by integrating both sides, and find the particular solution when an initial condition is given.

1. Find the general solution to the differential equation.

$$\frac{dy}{dx} = e^x - 6x^2$$

Answer: _____

2. Find the particular solution that passes through the point (1, 0).

$$\frac{dy}{dx} = e^x - 6x^2$$

Answer: _____

3. Find the general solution to the differential equation.

$$\frac{dy}{dx} = 4x^3 + 2x$$

Answer: _____

4. Find the general solution to the differential equation.

$$\frac{dy}{dx} = \sin(x) + \cos(x)$$

Answer: _____

5. Find the particular solution that passes through the point (0, 1).

$$\frac{dy}{dx} = 3x^2 - 4x$$

Answer: _____

6. Find the general solution to the differential equation.

$$\frac{dy}{dx} = \frac{1}{x} + e^{2x}$$

Answer: _____

Scan to watch

7. Find the particular solution that passes through the point (1, 2).

$$\frac{dy}{dx} = 6x^2 + 2$$

Answer: _____

8. Find the general solution to the differential equation.

$$\frac{dy}{dx} = \sqrt{x} + x^2$$

Answer: _____

9. Find the particular solution that passes through the point (0, 3).

$$\frac{dy}{dx} = e^x + 2x$$

Answer: _____

10. Find the general solution to the differential equation.

$$\frac{dy}{dx} = 5x^4 - 3x^2 + 7$$

Answer: _____

Evaluating Differential Equations

Calculus Worksheet · Grade 11-12

ANSWER KEY & SOLUTIONS

Remind students to include the constant of integration C for general solutions and to substitute initial conditions to find C for particular solutions.

Solutions

1. Find the general solution to the differential equation.

$$\frac{dy}{dx} = e^x - 6x^2$$

→ *Cross-multiply to separate variables: dy equals the right side times dx*
→ *Integrate both sides of the equation*
→ *The integral of dy is y, and integrate each term on the right side*
→ *Simplify the coefficient six divided by three to get two*
→ *Add the constant of integration C*

Answer: $y = e^x - 2x^3 + C$

---

2. Find the particular solution that passes through the point (1, 0).

$$\frac{dy}{dx} = e^x - 6x^2$$

→ *Find the general solution by integrating both sides*
→ *The general solution is y equals e to the x minus 2x cubed plus C*
→ *Substitute x equals 1 and y equals 0 into the equation*
→ *Solve 0 equals e minus 2 plus C for C*
→ *Find C equals negative e plus 2 and write the particular solution*

Answer: $y = e^x - 2x^3 - e + 2$

---

3. Find the general solution to the differential equation.

$$\frac{dy}{dx} = 4x^3 + 2x$$

→ *Separate variables by multiplying both sides by dx*
→ *Integrate both sides of the equation*
→ *Apply the power rule to each term*
→ *Add the constant of integration C*

Answer: $y = x^4 + x^2 + C$

---

4. Find the general solution to the differential equation.

$$\frac{dy}{dx} = \sin(x) + \cos(x)$$

→ *Cross-multiply dx to separate variables*
→ *Integrate both sides*
→ *The integral of sine x is negative cosine x*
→ *The integral of cosine x is sine x*
→ *Add the constant of integration C*

Answer: $y = -\cos(x) + \sin(x) + C$

Scan to watch

5. Find the particular solution that passes through the point (0, 1).

$$\frac{dy}{dx} = 3x^2 - 4x$$

→ *Integrate both sides to find the general solution*
→ *The general solution is y equals x cubed minus 2x squared plus C*
→ *Substitute x equals 0 and y equals 1*
→ *Solve for C to get C equals 1*
→ *Write the particular solution*

Answer: $y = x^3 - 2x^2 + 1$

---

6. Find the general solution to the differential equation.

$$\frac{dy}{dx} = \frac{1}{x} + e^{2x}$$

→ *Separate variables by multiplying both sides by dx*
→ *Integrate both sides*
→ *The integral of one over x is the natural log of absolute value of x*
→ *Use u-substitution for e to the 2x to get one-half e to the 2x*
→ *Add the constant of integration C*

Answer: $y = \ln|x| + \frac{1}{2}e^{2x} + C$

---

7. Find the particular solution that passes through the point (1, 2).

$$\frac{dy}{dx} = 6x^2 + 2$$

→ *Integrate both sides to find the general solution*
→ *The general solution is y equals 2x cubed plus 2x plus C*
→ *Substitute x equals 1 and y equals 2*
→ *Solve 2 equals 2 plus 2 plus C to find C equals negative 2*
→ *Write the particular solution*

Answer: $y = 2x^3 + 2x - 2$

---

8. Find the general solution to the differential equation.

$$\frac{dy}{dx} = \sqrt{x} + x^2$$

→ *Rewrite the square root as x to the one-half power*
→ *Cross-multiply dx to separate variables*
→ *Integrate both sides using the power rule*
→ *For x to the one-half, add one to the exponent and divide*
→ *Add the constant of integration C*

Answer: $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{x^3}{3} + C$

---

9. Find the particular solution that passes through the point (0, 3).

$$\frac{dy}{dx} = e^x + 2x$$

→ *Integrate both sides to find the general solution*
→ *The general solution is y equals e to the x plus x squared plus C*
→ *Substitute x equals 0 and y equals 3*
→ *Solve 3 equals 1 plus 0 plus C to find C equals 2*
→ *Write the particular solution*

Answer: $y = e^x + x^2 + 2$

---

10. Find the general solution to the differential equation.

$$\frac{dy}{dx} = 5x^4 - 3x^2 + 7$$

→ *Separate variables by multiplying both sides by dx*
→ *Integrate each term on the right side using the power rule*
→ *Simplify the coefficients*
→ *Add the constant of integration C*

Answer: $y = x^5 - x^3 + 7x + C$