Properties of Definite Integrals

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Apply the constant multiple and sum/difference properties of definite integrals
- Use the property of reversing limits of integration with a sign change
- Combine adjacent integration intervals using the additivity property

Use the given integral values and the properties of definite integrals to evaluate each expression below.

1. Given that the integral of f(x) from 0 to a equals 37 and the integral of g(x) from 0 to a equals 16, evaluate the following.

$$\int_0^a [2f(x) + 3g(x)]\,dx$$

Answer: _____

2. Given that the integral of f(x) from 1 to 4 equals -2, evaluate the following.

$$\int_4^1 f(x)\,dx$$

Answer: _____

3. Given that the integral of f(x) from -1 to 1 equals 5 and the integral of f(x) from 1 to 4 equals -2, evaluate the following.

$$\int_{-1}^4 f(x)\,dx$$

Answer: _____

4. Given that the integral of f(x) from 0 to a equals 37 and the integral of g(x) from 0 to a equals 16, evaluate the following.

$$\int_0^a [5f(x) - 2g(x)]\,dx$$

Answer: _____

5. Given that the integral of h(x) from -1 to 2 equals 7, evaluate the following.

$$\int_2^{-1} 4h(x)\,dx$$

Answer: _____

6. Given that the integral of f(x) from -1 to 1 equals 5 and the integral of f(x) from 1 to 4 equals -2, evaluate the following.

$$\int_{4}^{-1} f(x)\,dx$$

Answer: _____

7. Given that the integral of f(x) from 0 to a equals 37, evaluate the following.

$$\int_{0}^{a} \frac{1}{2} f(x)\,dx$$

Answer: _____

8. Given that the integral of f(x) from -1 to 1 equals 5 and the integral of h(x) from -1 to 2 equals 7, evaluate the following.

$$\int_{-1}^{1} f(x)\,dx + \int_{-1}^{2} 2h(x)\,dx$$

Answer: _____

9. Given that the integral of f(x) from -1 to 1 equals 5 and the integral of f(x) from 1 to 4 equals -2, evaluate the following.

$$\int_{-1}^{4} 3f(x)\,dx$$

Answer: _____

10. Given that the integral of f(x) from 0 to a equals 37 and the integral of g(x) from 0 to a equals 16, evaluate the following.

$$\int_{a}^{0} [f(x) + g(x)]\,dx$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students that constants can be factored out of integrals, swapping limits introduces a negative sign, and adjacent intervals can be combined via additivity.

Solutions

1. Given that the integral of f(x) from 0 to a equals 37 and the integral of g(x) from 0 to a equals 16, evaluate the following.

$$\int_0^a [2f(x) + 3g(x)]\,dx$$

→ *Split the integral into two separate integrals using the sum property.*
→ *Factor the constants 2 and 3 outside their respective integrals.*
→ *Substitute 37 for the integral of f(x) and 16 for the integral of g(x).*
→ *Compute 2 times 37 plus 3 times 16 to get 122.*

Answer: 122

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2. Given that the integral of f(x) from 1 to 4 equals -2, evaluate the following.

$$\int_4^1 f(x)\,dx$$

→ *Notice the lower limit is greater than the upper limit.*
→ *Apply the reversal property by swapping the limits and introducing a negative sign.*
→ *Replace the integral from 1 to 4 with -2.*
→ *Multiply negative one by negative two to obtain positive two.*

Answer: 2

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3. Given that the integral of f(x) from -1 to 1 equals 5 and the integral of f(x) from 1 to 4 equals -2, evaluate the following.

$$\int_{-1}^4 f(x)\,dx$$

→ *Use the additivity property to split the interval at x equals 1.*
→ *Write the integral as the integral from -1 to 1 plus the integral from 1 to 4.*
→ *Substitute 5 and -2 for the two integrals.*
→ *Add 5 and -2 to get 3.*

Answer: 3

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4. Given that the integral of f(x) from 0 to a equals 37 and the integral of g(x) from 0 to a equals 16, evaluate the following.

$$\int_0^a [5f(x) - 2g(x)]\,dx$$

→ *Apply the difference property to split the integral into two pieces.*
→ *Pull the constants 5 and -2 outside the integrals.*
→ *Substitute 37 for the integral of f(x) and 16 for the integral of g(x).*
→ *Compute 5 times 37 minus 2 times 16 to get 185 minus 32 equals 153.*

Answer: 153

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5. Given that the integral of h(x) from -1 to 2 equals 7, evaluate the following.

$$\int_{2}^{-1} 4h(x)\,dx$$

→ *Factor the constant 4 outside the integral.*
→ *Swap the limits of integration and introduce a negative sign.*
→ *Replace the integral from -1 to 2 with 7.*
→ *Compute -4 times 7 to get -28.*

Answer: -28

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6. Given that the integral of f(x) from -1 to 1 equals 5 and the integral of f(x) from 1 to 4 equals -2, evaluate the following.

$$\int_{4}^{-1} f(x)\,dx$$

→ *Reverse the limits of integration to write the integral from -1 to 4 with a negative sign.*
→ *Split the integral from -1 to 4 at x equals 1 using additivity.*
→ *Substitute 5 and -2 for the two component integrals.*
→ *The sum is 3, and applying the negative sign gives -3.*

Answer: -3

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7. Given that the integral of f(x) from 0 to a equals 37, evaluate the following.

$$\int_{0}^{a} \frac{1}{2} f(x)\,dx$$

→ *Factor the constant one-half outside the integral.*
→ *Substitute 37 for the integral of f(x) from 0 to a.*
→ *Multiply one-half by 37 to obtain thirty-seven halves.*

Answer: $\frac{37}{2}$

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8. Given that the integral of f(x) from -1 to 1 equals 5 and the integral of h(x) from -1 to 2 equals 7, evaluate the following.

$$\int_{-1}^{1} f(x)\,dx + \int_{-1}^{2} 2h(x)\,dx$$

→ *Evaluate the first integral by substituting its given value of 5.*
→ *Factor the constant 2 outside the second integral.*
→ *Substitute 7 for the integral of h(x) and compute 2 times 7 equals 14.*
→ *Add 5 and 14 to get 19.*

Answer: 19

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9. Given that the integral of f(x) from -1 to 1 equals 5 and the integral of f(x) from 1 to 4 equals -2, evaluate the following.

$$\int_{-1}^{4} 3f(x)\,dx$$

→ *Pull the constant 3 outside the integral.*
→ *Use additivity to combine the intervals from -1 to 1 and from 1 to 4.*
→ *Substitute 5 and -2 to get a combined integral value of 3.*
→ *Multiply 3 by 3 to obtain 9.*

Answer: 9

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10. Given that the integral of f(x) from 0 to a equals 37 and the integral of g(x) from 0 to a equals 16, evaluate the following.

$$\int_a^0 [f(x) + g(x)]\,dx$$

→ *Reverse the limits of integration and introduce a negative sign.*
→ *Split the resulting integral into the sum of two integrals from 0 to a.*
→ *Substitute 37 for the integral of f(x) and 16 for the integral of g(x).*
→ *Add 37 and 16 to get 53, then apply the negative sign to obtain -53.*

Answer: -53