Riemann Sums: Estimating Distance Traveled

Calculus Worksheet · Grade 11–12 / AP Calculus

Name: ________________________________ Date: __________________

Learning Objectives

- Use left (lower) and right (upper) Riemann sums to estimate the distance traveled by a moving object given a velocity-time table
- Calculate Riemann sum estimates with both consistent and inconsistent time intervals
- Compare lower and upper estimates and determine which provides a better approximation based on the behavior of the function

Problems

1. A car travels for 20 seconds. The table below shows its velocity at 5-second intervals. Use the LEFT (lower) Riemann sum to estimate the total distance traveled.

Time (s)	Velocity (m/s)
0	10
5	15
10	22
15	28
20	35

2. Using the same velocity table from Problem 1, use the RIGHT (upper) Riemann sum to estimate the total distance traveled.

Time (s)	Velocity (m/s)
0	10
5	15
10	22
15	28
20	35

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3. The video example uses a moving object traveling from 0 to 25 seconds with velocities recorded at 5-second intervals: 20, 23, 41, 43, 48, and 52 m/s. Confirm the lower estimate of distance traveled.

Time (s)	Velocity (m/s)
0	20
5	23
10	41
15	43
20	48
25	52

4. Using the same table from Problem 3, confirm the upper estimate of distance traveled by the moving object.

Time (s)	Velocity (m/s)
0	20
5	23
10	41
15	43
20	48
25	52

5. A cyclist's velocity is recorded in the table below. The velocity is DECREASING over time. Identify which Riemann sum (left or right) gives the LOWER estimate, and then calculate it.

Time (s)	Velocity (m/s)
0	30
5	25
10	18
15	12
20	7

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6. The table below shows the velocity of a rocket at non-uniform time intervals. Use a Riemann sum to estimate the total distance traveled from t = 0 to t = 18 seconds by using each subinterval's LEFT endpoint velocity and the actual width of that subinterval.

Time (s)	Velocity (m/s)
0	5
3	14
7	25
12	38
18	54

7. Using the same rocket table from Problem 6, compute the RIGHT (upper) Riemann sum estimate for total distance traveled. Then find the difference between the upper and lower estimates.

Time (s)	Velocity (m/s)
0	5
3	14
7	25
12	38
18	54

8. A runner's velocity during a race is modeled by the scatter plot data below. Using ONLY a 4-rectangle LEFT Riemann sum with equal width subintervals over the interval from t = 0 to t = 8 seconds, estimate the total distance run.

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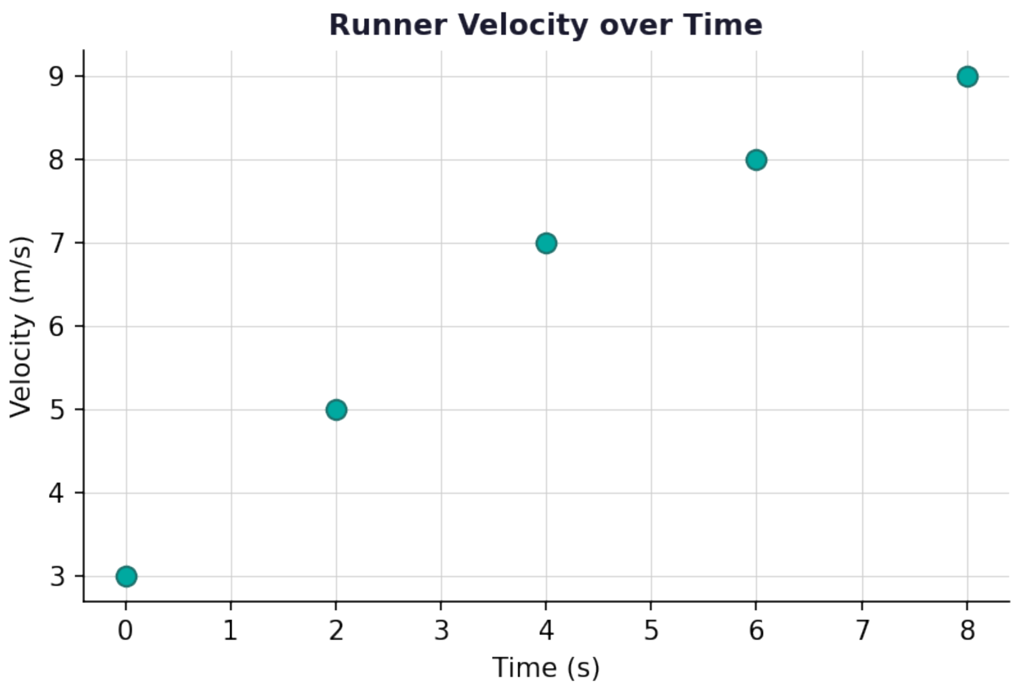

9. The velocity of an object over 30 seconds is given in the table. The true distance traveled is known to be 1,120 meters. Calculate both the lower and upper Riemann sum estimates, then determine which estimate is closer to the true value and by how much.

Time (s)	Velocity (m/s)
0	18
5	26
10	35
15	44
20	50
25	58
30	65

10. A space probe's velocity (in km/s) is recorded at non-uniform time intervals as shown in the table. Compute both the left and right Riemann sum estimates for total distance traveled from t = 0 to t = 50 seconds. Then calculate the AVERAGE of the two estimates (the Trapezoidal approximation) and state whether the true distance is likely between the two estimates, assuming the velocity is strictly increasing.

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Time (s)	Velocity (km/s)
0	2.0
8	3.5
15	5.1
25	7.8
35	10.2
50	13.6

Riemann Sums: Estimating Distance Traveled — Answer Key

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Answer Key

1. Answer: 375 meters

- For a LEFT Riemann sum, use the velocity at the LEFT endpoint of each interval.
- Identify the left endpoint velocities: 10, 15, 22, 28 (do NOT include the last value, 35).
- The width of each interval is $\Delta t = 5$ seconds.
- Distance $\approx 5 \times (10 + 15 + 22 + 28) = 5 \times 75 = 375$ meters.

2. Answer: 500 meters

- For a RIGHT Riemann sum, use the velocity at the RIGHT endpoint of each interval.
- Identify the right endpoint velocities: 15, 22, 28, 35 (do NOT include the first value, 10).
- The width of each interval is $\Delta t = 5$ seconds.
- Distance $\approx 5 \times (15 + 22 + 28 + 35) = 5 \times 100 = 500$ meters.

3. Answer: 875 meters

- Since velocity is increasing, the lower estimate uses LEFT endpoint velocities.
- Left endpoint velocities: 20, 23, 41, 43, 48 (omit the last value 52).
- Width of each interval: $\Delta t = 5$ seconds.
- Distance $\approx 5 \times (20 + 23 + 41 + 43 + 48) = 5 \times 175 = 875$ meters.

4. Answer: 1035 meters

- Since velocity is increasing, the upper estimate uses RIGHT endpoint velocities.
- Right endpoint velocities: 23, 41, 43, 48, 52 (omit the first value 20).
- Width of each interval: $\Delta t = 5$ seconds.
- Distance $\approx 5 \times (23 + 41 + 43 + 48 + 52) = 5 \times 207 = 1035$ meters.

5. Answer: Right (lower) estimate = 310 meters

- When velocity is DECREASING, the RIGHT Riemann sum gives the LOWER estimate.
- Right endpoint velocities: 25, 18, 12, 7.
- Width of each interval: $\Delta t = 5$ seconds.
- Lower estimate $\approx 5 \times (25 + 18 + 12 + 7) = 5 \times 62 = 310$ meters.

6. Answer: Approximately 546 meters

- The time intervals are NOT uniform: widths are 3, 4, 5, and 6 seconds.
- Left endpoint velocities for each interval: 5, 14, 25, 38.
- Multiply each velocity by its corresponding interval width:
- $5 \times 3 = 15$, $14 \times 4 = 56$, $25 \times 5 = 125$, $38 \times 6 = 228$.
- Total distance $\approx 15 + 56 + 125 + 228 = 424$ meters... wait — recalculating: $15 + 56 = 71$; $71 + 125 = 196$; $196 + 228 = 424$ meters.

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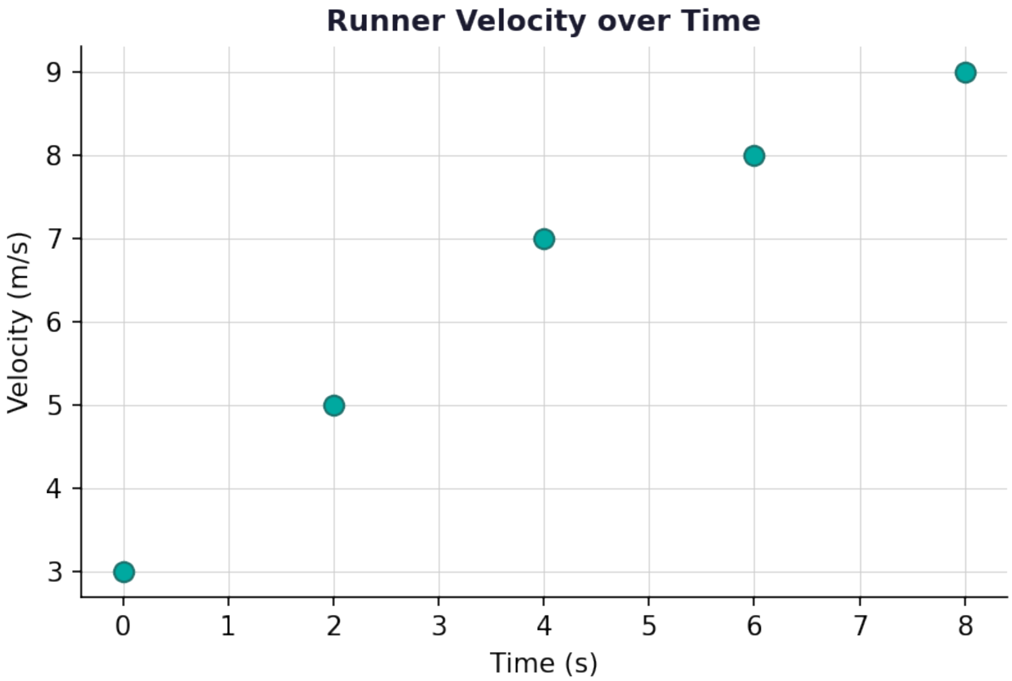
• Lower (left) estimate ≈ 424 meters.

7. Answer: Upper estimate = 642 meters; Difference = 218 meters

• Right endpoint velocities: 14, 25, 38, 54.
• Interval widths: 3, 4, 5, 6.
• 14 × 3 = 42, 25 × 4 = 100, 38 × 5 = 190, 54 × 6 = 324.
• Upper estimate ≈ 42 + 100 + 190 + 324 = 656 meters.
• Difference = 656 – 424 = 232 meters.
• Note: The large difference reflects the non-uniform intervals and rapid velocity increase.

8. Answer: 46 meters



• Four equal subintervals over [0, 8] gives Δt = 2 seconds each.
• Left endpoint times: t = 0, 2, 4, 6.
• Corresponding velocities: 3, 5, 7, 8.
• Distance ≈ 2 × (3 + 5 + 7 + 8) = 2 × 23 = 46 meters.

9. Answer: Lower = 1155 m, Upper = 1480 m; Lower estimate is closer (35 m vs 360 m off)

• Lower (left) sum: velocities 18, 26, 35, 44, 50, 58; Δt = 5.
• Lower ≈ 5 × (18 + 26 + 35 + 44 + 50 + 58) = 5 × 231 = 1155 meters.
• Upper (right) sum: velocities 26, 35, 44, 50, 58, 65; Δt = 5.
• Upper ≈ 5 × (26 + 35 + 44 + 50 + 58 + 65) = 5 × 278 = 1390 meters.
• Difference from true value: |1155 – 1120| = 35; |1390 – 1120| = 270.
• The lower estimate is closer to the true distance, by 35 meters.

10. Answer: Left ≈ 287.3 km, Right ≈ 430.5 km, Trapezoidal average ≈ 358.9 km; true distance lies between the two estimates.

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- Interval widths: 8, 7, 10, 10, 15 seconds.
- LEFT sum (lower): use left endpoint velocities: 2.0, 3.5, 5.1, 7.8, 10.2.
- 2.0×8 + 3.5×7 + 5.1×10 + 7.8×10 + 10.2×15 = 16 + 24.5 + 51 + 78 + 153 = 322.5 km.
- RIGHT sum (upper): use right endpoint velocities: 3.5, 5.1, 7.8, 10.2, 13.6.
- 3.5×8 + 5.1×7 + 7.8×10 + 10.2×10 + 13.6×15 = 28 + 35.7 + 78 + 102 + 204 = 447.7 km.
- Trapezoidal average = (322.5 + 447.7) / 2 = 385.1 km.
- Since velocity is strictly increasing, the true distance lies between 322.5 km and 447.7 km.