Separable Differential Equations

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 10

Learning Objectives

- Separate variables in a first-order differential equation
- Integrate both sides to find the general solution
- Apply initial conditions to determine the particular solution

Solve each separable differential equation using the initial condition to find the particular solution for y.

1. Find the equation of the curve passing through (0, 7) with the given slope.

$$\frac{dy}{dx} = 4x^3y$$

Answer: _____

2. Solve the separable differential equation with the given initial condition.

$$\frac{dy}{dx} = 2xy,\ y(0) = 3$$

Answer: _____

3. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = 6x^2y,\ y(0) = 5$$

Answer: _____

4. Solve the differential equation with the initial point given.

$$\frac{dy}{dx} = \frac{y}{x},\ y(1) = 4$$

Answer: _____

5. Find y as a function of x.

$$\frac{dy}{dx} = 3x^2y,\ y(0) = 2$$

Answer: _____

6. Determine the curve passing through the given point.

$$\frac{dy}{dx} = -2xy,\ y(0) = 10$$

Answer: _____

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7. Solve the separable equation using the initial condition.

$$\frac{dy}{dx} = 5y,\ y(0) = 1$$

Answer: _____

8. Find the particular solution of the differential equation.

$$\frac{dy}{dx} = 8x^3y,\ y(0) = 6$$

Answer: _____

9. Solve the differential equation with the given condition.

$$\frac{dy}{dx} = x^2y,\ y(0) = 4$$

Answer: _____

10. Find the function y satisfying the equation.

$$\frac{dy}{dx} = \frac{4x^3}{y},\ y(0) = 2$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Emphasize the three-step process: separate variables, integrate both sides, then apply the initial condition to solve for the constant C.

Solutions

1. Find the equation of the curve passing through (0, 7) with the given slope.

$$\frac{dy}{dx} = 4x^3y$$

→ *Separate variables to get dy over y equals 4x cubed dx.*
→ *Integrate both sides to get ln of absolute value of y equals x to the fourth plus C.*
→ *Substitute x equals 0 and y equals 7 to find C equals ln 7.*
→ *Exponentiate both sides and simplify to get y equals 7 times e to the x to the fourth.*

Answer: $y = 7e^{x^4}$

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2. Solve the separable differential equation with the given initial condition.

$$\frac{dy}{dx} = 2xy,\ y(0) = 3$$

→ *Separate variables to get dy over y equals 2x dx.*
→ *Integrate both sides to obtain ln of absolute value of y equals x squared plus C.*
→ *Apply the initial condition y of 0 equals 3 to find C equals ln 3.*
→ *Exponentiate both sides to get y equals 3 times e to the x squared.*

Answer: $y = 3e^{x^2}$

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3. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = 6x^2y,\ y(0) = 5$$

→ *Separate variables to get dy over y equals 6x squared dx.*
→ *Integrate both sides to get ln of absolute value of y equals 2x cubed plus C.*
→ *Use y of 0 equals 5 to determine C equals ln 5.*
→ *Exponentiate both sides to obtain y equals 5 times e to the 2x cubed.*

Answer: $y = 5e^{2x^3}$

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4. Solve the differential equation with the initial point given.

$$\frac{dy}{dx} = \frac{y}{x},\ y(1) = 4$$

→ *Separate variables to obtain dy over y equals dx over x.*
→ *Integrate both sides to get ln of absolute value of y equals ln of absolute value of x plus C.*
→ *Substitute x equals 1 and y equals 4 to find C equals ln 4.*
→ *Exponentiate and simplify to get y equals 4x.*

Answer: $y = 4x$

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5. Find y as a function of x.

$$\frac{dy}{dx} = 3x^2y,\ y(0) = 2$$

→ *Separate variables to get dy over y equals 3x squared dx.*
→ *Integrate both sides to obtain ln of absolute value of y equals x cubed plus C.*
→ *Apply y of 0 equals 2 to determine C equals ln 2.*
→ *Exponentiate both sides to get y equals 2 times e to the x cubed.*

Answer: $y = 2e^{x^3}$

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6. Determine the curve passing through the given point.

$$\frac{dy}{dx} = -2xy,\ y(0) = 10$$

→ *Separate variables to get dy over y equals negative 2x dx.*
→ *Integrate both sides to obtain ln of absolute value of y equals negative x squared plus C.*
→ *Use y of 0 equals 10 to find C equals ln 10.*
→ *Exponentiate both sides to get y equals 10 times e to the negative x squared.*

Answer: $y = 10e^{-x^2}$

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7. Solve the separable equation using the initial condition.

$$\frac{dy}{dx} = 5y,\ y(0) = 1$$

→ *Separate variables to obtain dy over y equals 5 dx.*
→ *Integrate both sides to get ln of absolute value of y equals 5x plus C.*
→ *Apply y of 0 equals 1 to find C equals 0.*
→ *Exponentiate both sides to get y equals e to the 5x.*

Answer: $y = e^{5x}$

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8. Find the particular solution of the differential equation.

$$\frac{dy}{dx} = 8x^3y,\ y(0) = 6$$

→ *Separate variables to obtain dy over y equals 8x cubed dx.*
→ *Integrate both sides to get ln of absolute value of y equals 2x to the fourth plus C.*
→ *Use y of 0 equals 6 to determine C equals ln 6.*
→ *Exponentiate both sides to obtain y equals 6 times e to the 2x to the fourth.*

Answer: $y = 6e^{2x^4}$

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9. Solve the differential equation with the given condition.

$$\frac{dy}{dx} = x^2y,\ y(0) = 4$$

→ *Separate variables to obtain dy over y equals x squared dx.*
→ *Integrate both sides to get ln of absolute value of y equals x cubed over 3 plus C.*
→ *Apply y of 0 equals 4 to determine C equals ln 4.*
→ *Exponentiate both sides to get y equals 4 times e to the x cubed over 3.*

Answer: $y = 4e^{\frac{x^3}{3}}$

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10. Find the function y satisfying the equation.

$$\frac{dy}{dx} = \frac{4x^3}{y},\ y(0) = 2$$

→ *Separate variables to obtain y dy equals 4x cubed dx.*
→ *Integrate both sides to get y squared over 2 equals x to the fourth plus C.*
→ *Apply y of 0 equals 2 to find C equals 2.*
→ *Multiply by 2 and take the square root to get y equals the square root of 2x to the fourth plus 4.*

Answer: $y = \sqrt{2x^4 + 4}$