Integration by Substitution (U-Substitution)

Calculus Worksheet · Grade 11–12 / College Intro Calculus

Name: ________________________________ Date: __________________

Learning Objectives

- Identify the appropriate substitution (u) for a composite integrand
- Rewrite an integral entirely in terms of u and du, adjusting for constant factors
- Integrate using the Power Rule and back-substitute to express the answer in terms of x

Problems

1. Use the substitution rule to find the integral of 3x squared times the square root of x cubed plus 5, with respect to x. Let u equal x cubed plus 5.

$$\int 3x^2\sqrt{x^3+5}\,dx$$

2. Find the integral of 4x times the quantity x squared plus 7, raised to the third power, with respect to x. Let u equal x squared plus 7.

$$\int 4x(x^2+7)^3\,dx$$

3. Use u-substitution to evaluate the integral of 5x to the fourth power times the quantity x to the fifth minus 2, raised to the fourth power, with respect to x.

$$\int 5x^4(x^5-2)^4\,dx$$

4. Find the integral of x cubed times the quantity x to the fourth plus 1, raised to the fifth power, with respect to x. Note that you will need to adjust for a constant factor.

$$\int x^3(x^4+1)^5\,dx$$

5. Use u-substitution to evaluate the integral of secant squared of 3x, with respect to x.

$$\int \sec^2(3x)\,dx$$

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6. Find the integral of secant squared of the quantity 1 over x, all divided by x squared, with respect to x. Let u equal 1 over x.

7. Evaluate the integral of 6x to the fifth power divided by the square root of x to the sixth plus 4, with respect to x.

$$\int \frac{6x^5}{\sqrt{x^6 + 4}}\, dx$$

8. Use u-substitution to find the integral of x squared times the quantity 2 plus x cubed, raised to the negative second power, with respect to x.

$$\int x^2(2 + x^3)^{-2}\, dx$$

9. Evaluate the integral of the quantity 3x squared minus 2 times the quantity x cubed minus 2x, raised to the seventh power, with respect to x.

$$\int (3x^2 - 2)(x^3 - 2x)^7\, dx$$

10. Find the integral of x squared times the square root of the quantity 4 minus x cubed, raised to the fifth power, with respect to x. (Hint: rewrite the radical as a fractional exponent and adjust for constant factors.)

$$\int x^2(4 - x^3)^{5/2}\, dx$$

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Integration by Substitution (U-Substitution) — Answer Key

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Answer Key

1. Answer: (2/3)(x^3 + 5)^(3/2) + C

- Let u = x^3 + 5, so du = $3x^2$ dx
- Rewrite: ∫ √u du
- Rewrite as: ∫ u^(1/2) du
- Integrate: u^(3/2) / (3/2) + C = (2/3)u^(3/2) + C
- Back-substitute: (2/3)(x^3 + 5)^(3/2) + C

2. Answer: (x^2 + 7)■ / 2 + C

- Let u = x^2 + 7, so du = 2x dx, meaning 4x dx = 2 du
- Rewrite: ∫ $2u^3$ du
- Integrate: 2 · u■/4 + C = u■/2 + C
- Back-substitute: (x^2 + 7)■ / 2 + C

3. Answer: (x■ – 2)■ / 5 + C

- Let u = x■ – 2, so du = 5x■ dx
- Rewrite: ∫ u■ du
- Integrate: u■/5 + C
- Back-substitute: (x■ – 2)■ / 5 + C

4. Answer: (x■ + 1)■ / 24 + C

- Let u = x■ + 1, so du = $4x^3$ dx, meaning x^3 dx = (1/4) du
- Rewrite: ∫ (1/4) u■ du
- Integrate: (1/4) · u■/6 + C = u■/24 + C
- Back-substitute: (x■ + 1)■ / 24 + C

5. Answer: (1/3) tan(3x) + C

- Let u = 3x, so du = 3 dx, meaning dx = (1/3) du
- Rewrite: ∫ (1/3) $\sec^2$(u) du
- Integrate: (1/3) tan(u) + C
- Back-substitute: (1/3) tan(3x) + C

6. Answer: –tan(1/x) + C

- Let u = 1/x = x^(–1), so du = –x^(–2) dx = –(1/x^2) dx
- Therefore (1/x^2) dx = –du
- Rewrite: ∫ $\sec^2$(u) · (–du) = –∫ $\sec^2$(u) du
- Integrate: –tan(u) + C
- Back-substitute: –tan(1/x) + C

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7. Answer: 2√(x■ + 4) + C

- Let u = x■ + 4, so du = 6x■ dx
- Rewrite: ∫ u^(–1/2) du
- Integrate: u^(1/2) / (1/2) + C = 2u^(1/2) + C
- Back-substitute: 2√(x■ + 4) + C

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8. Answer: –1 / (3(2 + x^3)) + C

- Let u = 2 + x^3, so du = 3x^2 dx, meaning x^2 dx = (1/3) du
- Rewrite: ∫ (1/3) u^(–2) du
- Integrate: (1/3) · u^(–1)/(–1) + C = –1/(3u) + C
- Back-substitute: –1 / (3(2 + x^3)) + C

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9. Answer: (x^3 – 2x)■ / 8 + C

- Let u = x^3 – 2x, so du = (3x^2 – 2) dx
- Rewrite: ∫ u■ du
- Integrate: u■/8 + C
- Back-substitute: (x^3 – 2x)■ / 8 + C

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10. Answer: –2(4 – x^3)^(7/2) / 21 + C

- Let u = 4 – x^3, so du = –3x^2 dx, meaning x^2 dx = –(1/3) du
- Rewrite: ∫ u^(5/2) · (–1/3) du = –(1/3) ∫ u^(5/2) du
- Integrate: –(1/3) · u^(7/2) / (7/2) + C = –(1/3) · (2/7) u^(7/2) + C
- Simplify: –2u^(7/2)/21 + C
- Back-substitute: –2(4 – x^3)^(7/2) / 21 + C

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