Volumes of Solids of Revolution (Disk Method)

Calculus Worksheet · Grade 11-12

Name: _____ Date: _____ Score: / 9

Learning Objectives

- Apply the disk method to compute volumes of solids of revolution about the x-axis
- Set up definite integrals representing volumes generated by rotating a region about a given axis
- Evaluate volume integrals using the formula V equals the integral of pi times R squared dx

For each problem, set up the definite integral using the disk method and evaluate to find the exact volume of the solid of revolution.

1. Find the volume of the solid obtained by rotating the region under y = sqrt(x) from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(\sqrt{x})^2\,dx$$

Answer: _____

2. Find the volume of the solid generated by rotating y = x^{2} from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(x^2)^2\,dx$$

Answer: _____

3. Compute the volume of the solid obtained by rotating y = x from x = 0 to x = 2 about the x-axis.

$$V = \int_0^2 \pi(x)^2\,dx$$

Answer: _____

4. Find the volume generated by rotating y = sqrt(x) from x = 1 to x = 4 about the x-axis.

$$V = \int_1^4 \pi(\sqrt{x})^2\,dx$$

Answer: _____

5. Find the volume of the solid obtained by rotating y = x^{3} from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(x^3)^2\,dx$$

Answer: _____

6. Compute the volume of the solid obtained by rotating y = 2x from x = 0 to x = 3 about the x-axis.

$$V = \int_0^3 \pi(2x)^2\,dx$$

Answer: _____

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7. Find the volume generated by rotating y = sqrt(x) + 1 from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(\sqrt{x} + 1)^2 \, dx$$

Answer: _____

8. Find the volume of the solid obtained by rotating y = e^{x} from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(e^x)^2 \, dx$$

Answer: _____

9. Compute the volume of the solid obtained by rotating y = 1 over x from x = 1 to x = 2 about the x-axis.

$$V = \int_1^2 \pi\left(\frac{1}{x}\right)^2 \, dx$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Remind students to sketch the region and identify the radius R(x) as the top function minus the axis of rotation before integrating.

Solutions

1. Find the volume of the solid obtained by rotating the region under y = sqrt(x) from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(\sqrt{x})^2\,dx$$

→ *Identify the radius R of x as the square root of x*
→ *Square the radius to get x*
→ *Integrate pi times x from 0 to 1 to get pi times one half*
→ *Simplify to pi over 2*

Answer: $\frac{\pi}{2}$

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2. Find the volume of the solid generated by rotating y = x^{2} from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(x^2)^2\,dx$$

→ *Square the radius x squared to get x to the fourth*
→ *Integrate pi times x to the fourth from 0 to 1*
→ *Evaluate to get pi times one fifth*
→ *Simplify to pi over 5*

Answer: $\frac{\pi}{5}$

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3. Compute the volume of the solid obtained by rotating y = x from x = 0 to x = 2 about the x-axis.

$$V = \int_0^2 \pi(x)^2\,dx$$

→ *Square the radius x to get x squared*
→ *Integrate pi times x squared from 0 to 2*
→ *Apply the power rule to get pi times x cubed over 3*
→ *Evaluate from 0 to 2 to get 8 pi over 3*

Answer: $\frac{8\pi}{3}$

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4. Find the volume generated by rotating y = sqrt(x) from x = 1 to x = 4 about the x-axis.

$$V = \int_1^4 \pi(\sqrt{x})^2\,dx$$

→ *Square the radius to get x*
→ *Integrate pi times x from 1 to 4*
→ *Evaluate pi times x squared over 2 at the bounds*
→ *Subtract to get 15 pi over 2*

Answer: $\frac{15\pi}{2}$

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5. Find the volume of the solid obtained by rotating y = x^{3} from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(x^3)^2\,dx$$

→ *Square the radius x cubed to get x to the sixth*
→ *Integrate pi times x to the sixth from 0 to 1*
→ *Apply the power rule to get pi times x to the seventh over 7*
→ *Evaluate to get pi over 7*

Answer: $\frac{\pi}{7}$

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6. Compute the volume of the solid obtained by rotating y = 2x from x = 0 to x = 3 about the x-axis.

$$V = \int_0^3 \pi(2x)^2\,dx$$

→ *Square the radius 2x to get 4 x squared*
→ *Integrate pi times 4 x squared from 0 to 3*
→ *Apply the power rule to get 4 pi times x cubed over 3*
→ *Evaluate at 3 to get 36 pi*

Answer: 36π

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7. Find the volume generated by rotating y = sqrt(x) + 1 from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(\sqrt{x}+1)^2\,dx$$

→ *Expand the squared radius to get x plus 2 square root of x plus 1*
→ *Integrate pi times that expression from 0 to 1*
→ *Find antiderivatives x squared over 2 plus 4 x to the three halves over 3 plus x*
→ *Evaluate at the bounds to get 17 pi over 6*

Answer: $\frac{17\pi}{6}$

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8. Find the volume of the solid obtained by rotating y = e^{x} from x = 0 to x = 1 about the x-axis.

$$V = \int_0^1 \pi(e^x)^2\,dx$$

→ *Square the radius e to the x to get e to the 2 x*
→ *Integrate pi times e to the 2 x from 0 to 1*
→ *The antiderivative is pi times e to the 2 x over 2*
→ *Evaluate to get pi times the quantity e squared minus 1 over 2*

Answer: $\frac{\pi(e^2-1)}{2}$

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9. Compute the volume of the solid obtained by rotating y = 1 over x from x = 1 to x = 2 about the x-axis.

$$V = \int_1^2 \pi\left(\frac{1}{x}\right)^2\,dx$$

→ *Square the radius 1 over x to get 1 over x squared*
→ *Integrate pi times x to the negative 2 from 1 to 2*
→ *The antiderivative is negative pi over x*
→ *Evaluate to get negative pi over 2 plus pi which equals pi over 2*

Answer: $\frac{\pi}{2}$