Volumes Using Washers

Numberbender | WORKSHEET

Name: _____ Date: _____ Score: _____

Find the volume of each region using the Washer Method. Show all work.

Calculus 1 Worksheet #50
Volumes Using Washers

Notes: Today you learn how to find volumes using WASHERS: $\text{Volume} = \pi \int_a^b R^2 - r^2$

Medium (Washer Method)

Find the volume of $y = x^2$, $y = 4$ revolved around x-axis.

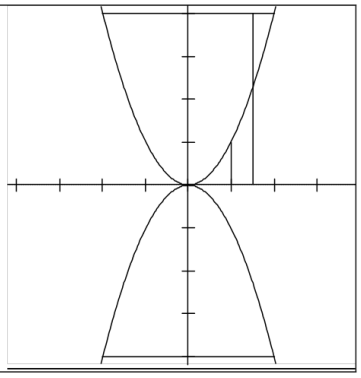
Cross sections are **circular washers**.
[A washer is a circle with a **hole** in its center.]
"Thickness" of **this** washer is $x_2 - x_1$, or **dx**.

$$\therefore \int_{x=}^{x=} \pi R^2 dx - \int_{x=}^{x=} \pi r^2 dx$$

[Volume **entire**] − [Volume **"hole"**] or

$$\pi \int_{x=-2}^{x=2} \left(R^2 - r^2\right) dx \Rightarrow \pi \int_{-2}^{2} \left[4^2 - y^2\right] dx \Rightarrow \pi \int_{-2}^{2} \left[16 - \left(x^2\right)^2\right] dx = \pi \int_{-2}^{2} \left(16 - x^4\right) dx$$

$$\pi \left[16x - \frac{x^5}{5}\right]_{-2}^{2} = \pi \left[\left(32 - \frac{32}{5}\right) - \left(-32 - \frac{-32}{5}\right)\right] = \pi \left[\frac{128}{5} - \left(-\frac{128}{5}\right)\right] = \boxed{\frac{256\pi}{5}}$$



Find the volume of the following:

1. $y = x^3$, $x = 2$, $y = 0$ (x–axis)
2. $y = 2x^2$, $y = 0$, $x = 5$ (x–axis)
3. $y = 4x^2$, $y = 16$ (y–axis)
4. $y = \frac{x}{2}$, $y = 2$ (y–axis)
5. $y = \sqrt{x}$, $x = 0$, $y = 2$ (y–axis)
6*. $y = \frac{x}{2}$, $x = 0$, $y = 2$ (x–axis)
7*. $y = \sqrt{x}$, $x = 0$, $y = 2$ (x–axis)
8*. $y = x$, $y = 3x - x^2$ (x–axis)
9*. $y = \frac{x^2}{3}$, $y = x$ (y- axis)

10. Use the Fundamental Theorem of Calculus to evaluate the integral: $\int_0^{\frac{\pi}{4}} \tan x dx =$

11. Use the Fundamental Theorem of Calculus to evaluate the integral: $\int_{\frac{\pi}{4}}^{5\pi/4} \sin x dx$

12. Calculate the area bounded by $y = x^3 - 4x + 4$ and $y = 3x - 2$.

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Answer key — for instructor use only.

[Use your calculator to sketch (CAREFULLY) and to find intersection points] **Do the rest by hand!**

Answers:

1. $\frac{128\pi}{7}u^3$	2. $2500\pi u^3$	3. $32\pi u^3$	4. $\frac{32\pi}{3}u^3$	5. $\frac{32\pi}{5}u^3$	6. $\frac{32\pi}{3}u^3$
7. $8\pi u^3$	8. $\frac{56\pi}{15}u^3$	9. $\frac{9\pi}{2}u^3$	10. $\ln\sqrt{2}$	11. $\sqrt{2}$	12. $\frac{131}{4}u^2$