



Evaluating Limits Using Algebraic Techniques

Calculus Worksheet · Grade 11-12

Name: _____

Date: _____

Score: / 9

Learning Objectives

- Evaluate limits using direct substitution and identify indeterminate forms
- Apply factoring techniques to simplify rational functions and evaluate limits
- Use rationalization with conjugates to evaluate limits involving square roots

Evaluate each limit using direct substitution, factoring, or rationalization as appropriate, and show all work.

1. Evaluate the limit by factoring.

$$\lim_{x \rightarrow 4} \frac{x-4}{x^2 - 7x + 12}$$

Answer: _____

2. Evaluate the limit by rationalizing the numerator.

$$\lim_{x \rightarrow 7} \frac{\sqrt{x-6} - 1}{7-x}$$

Answer: _____

3. Evaluate the limit by factoring.

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

Answer: _____

4. Evaluate the limit by factoring.

$$\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2}$$

Answer: _____

5. Evaluate the limit by rationalizing.

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$$

Answer: _____

6. Evaluate the limit using direct substitution.

$$\lim_{x \rightarrow 5} (3x^2 - 2x + 1)$$

Answer: _____



7. Evaluate the limit by factoring.

$$\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x + 2}$$

Answer: _____

8. Evaluate the limit by rationalizing the numerator.

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

Answer: _____

9. Evaluate the limit by factoring.

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

Answer: _____





Encourage students to always attempt direct substitution first, then choose factoring or rationalization based on the function's structure.

Solutions

1. Evaluate the limit by factoring.

$$\lim_{x \rightarrow 4} \frac{x-4}{x^2 - 7x + 12}$$

- Substitute x equals 4 to check, getting zero over zero, which is indeterminate
- Factor the denominator into x minus 3 times x minus 4
- Cancel the common factor x minus 4 from numerator and denominator
- Substitute x equals 4 into 1 over x minus 3 to get 1 over 1 equals 1

Answer: 1

2. Evaluate the limit by rationalizing the numerator.

$$\lim_{x \rightarrow 7} \frac{\sqrt{x-6} - 1}{7-x}$$

- Substitute x equals 7 to confirm the indeterminate form zero over zero
- Multiply numerator and denominator by the conjugate square root of x minus 6 plus 1
- Simplify the numerator to x minus 7
- Factor out negative one from 7 minus x to get negative of x minus 7
- Cancel x minus 7 from numerator and denominator
- Substitute x equals 7 into negative 1 over square root of 1 plus 1 to get negative one half

Answer: $-\frac{1}{2}$

3. Evaluate the limit by factoring.

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

- Substitute x equals 3 to check, getting zero over zero
- Factor the numerator as a difference of squares into x minus 3 times x plus 3
- Cancel the common factor x minus 3
- Substitute x equals 3 into x plus 3 to get 6

Answer: 6

4. Evaluate the limit by factoring.

$$\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2}$$

- Direct substitution yields zero over zero, indeterminate
- Factor the numerator as x minus 2 times x minus 3
- Cancel x minus 2 from numerator and denominator
- Substitute x equals 2 into x minus 3 to get negative one

Answer: -1



5. Evaluate the limit by rationalizing.

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$$

- Substitute x equals 0 to confirm the indeterminate form zero over zero
- Multiply numerator and denominator by the conjugate square root of x plus 4 plus 2
- Simplify the numerator using difference of squares to get x
- Cancel x from numerator and denominator
- Substitute x equals 0 into 1 over square root of 4 plus 2 to get one fourth

Answer: $\frac{1}{4}$

6. Evaluate the limit using direct substitution.

$$\lim_{x \rightarrow 5} (3x^2 - 2x + 1)$$

- Since the function is a polynomial, direct substitution works
- Substitute x equals 5 into $3x$ squared minus $2x$ plus 1
- Compute 3 times 25 minus 2 times 5 plus 1 equals 75 minus 10 plus 1
- Simplify to get 66

Answer: 66

7. Evaluate the limit by factoring.

$$\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x + 2}$$

- Direct substitution yields zero over zero
- Factor the numerator as x plus 2 times x plus 3
- Cancel the common factor x plus 2
- Substitute x equals negative 2 into x plus 3 to get 1

Answer: 1

8. Evaluate the limit by rationalizing the numerator.

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

- Direct substitution gives zero over zero, indeterminate
- Multiply numerator and denominator by the conjugate square root of x plus 3
- Simplify the numerator to x minus 9
- Cancel x minus 9 from numerator and denominator
- Substitute x equals 9 into 1 over square root of 9 plus 3 to get one sixth

Answer: $\frac{1}{6}$

9. Evaluate the limit by factoring.

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

- Direct substitution gives zero over zero
- Factor the numerator as a difference of cubes into x minus 1 times x squared plus x plus 1
- Cancel the common factor x minus 1
- Substitute x equals 1 into x squared plus x plus 1 to get 3

Answer: 3

