Translating Compound Statements: Symbolic ↔ English

Logic & Reasoning Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Translate symbolic compound statements (using ∧, ∨, ¬, →, ↔) into English sentences
- Translate English compound statements into symbolic form using correct logical operators
- Apply negation distribution and identify dominant operations in complex compound statements

Problems

1. Let P = 'It is raining' and Q = 'It is cold.' Translate the following symbolic statement into English.

2. Let P = 'She is tired' and Q = 'She is hungry.' Translate the following symbolic statement into English.

3. Let P = 'The sun is shining.' Translate the following symbolic statement into English.

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4. Let P = 'He is wealthy' and Q = 'He is happy.' Translate the following English statement into symbolic form: 'He is not wealthy and he is not happy.'

5. Let P = 'She is wealthy' and Q = 'She is happy.' Translate the following symbolic statement into English.

6. Let P = 'She is wealthy' and Q = 'She is happy.' Distribute the negation over the entire parenthetical expression and translate the result into English.

7. Let P = 'She is wealthy' and Q = 'She is happy.' Distribute the negation and translate the result into English.

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8. Let P = 'A student misses a lecture', Q = 'A student studies', and R = 'A student fails.' Translate the following symbolic compound statement into English.

9. Let P = 'A student misses a lecture', Q = 'A student studies', and R = 'A student fails.' Translate the following English statement into symbolic form: 'A student studies, and if a student does not miss a lecture then a student does not fail.'

10. Let P = 'A student misses a lecture', Q = 'A student studies', and R = 'A student fails.' Translate the following complex symbolic compound statement into English, identifying the dominant operation first.

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Translating Compound Statements: Symbolic ↔ English — Answer Key

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Answer Key

1. Answer: It is raining and it is cold.
- The symbol ∧ represents the conjunction 'and'.
- P = 'It is raining', Q = 'It is cold'
- P ∧ Q translates to: It is raining and it is cold.

2. Answer: She is tired or she is hungry.
- The symbol ∨ represents the disjunction 'or'.
- P = 'She is tired', Q = 'She is hungry'
- P ∨ Q translates to: She is tired or she is hungry.

3. Answer: The sun is not shining.
- The symbol ~ (or ¬) represents negation, meaning 'not'.
- P = 'The sun is shining'
- ~P translates to: The sun is not shining.

4. Answer: ~P ∧ ~Q
- Identify each simple statement: P = 'He is wealthy', Q = 'He is happy'
- Negate P: ~P = 'He is not wealthy'
- Negate Q: ~Q = 'He is not happy'
- Join with 'and' (∧): ~P ∧ ~Q

5. Answer: She is not wealthy and she is happy.
- ~P negates only P: 'She is not wealthy'
- Q remains unchanged: 'She is happy'
- ∧ connects them with 'and'
- Result: She is not wealthy and she is happy.

6. Answer: She is neither wealthy nor happy. (Symbolic: ~P ∧ ~Q)
- Distribute ~ over the disjunction: ~(P ∨ Q) = ~P ∧ ~Q (De Morgan's Law)
- ~P = 'She is not wealthy', ~Q = 'She is not happy'
- The conjunction of two negations can be phrased as 'neither...nor...'
- Result: She is neither wealthy nor happy.

7. Answer: She is not wealthy or she is not happy. (Symbolic: ~P ∨ ~Q)
- Apply De Morgan's Law: ~(P ∧ Q) = ~P ∨ ~Q
- ~P = 'She is not wealthy', ~Q = 'She is not happy'
- ∨ connects them with 'or'

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- Result: She is not wealthy or she is not happy.

8. Answer: If a student studies and does not miss a lecture, then the student does not fail.

- Identify the dominant operation: the conditional arrow (→) separates antecedent and consequent.
- Antecedent: Q ∧ ~P = 'A student studies and does not miss a lecture'
- Consequent: ~R = 'A student does not fail'
- Combine using 'if...then...': If a student studies and does not miss a lecture, then the student does not fail.

9. Answer: Q ∧ (~P → ~R)

- Identify the dominant operation: 'and' (∧) connects two parts.
- First part: 'A student studies' = Q
- Second part: 'if a student does not miss a lecture then a student does not fail' = ~P → ~R
- Combine: Q ∧ (~P → ~R)

10. Answer: A student misses a lecture or studies if and only if a student does not fail and does not miss a lecture.

- Identify the dominant operation: biconditional (↔) — 'if and only if'
- Left side: P ∨ Q = 'A student misses a lecture or a student studies'
- Right side: ~R ∧ ~P = 'A student does not fail and does not miss a lecture'
- Combine using 'if and only if': A student misses a lecture or studies if and only if a student does not fail and does not miss a lecture.

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