Deductive vs. Inductive Reasoning & The Law of Detachment

Geometry / Logic Worksheet · Grade 8–10

Name: _____ Date: _____

Learning Objectives

- Distinguish between inductive and deductive reasoning
- Identify conjectures formed through inductive reasoning
- Apply the Law of Detachment to determine valid conclusions

Problems

1. Identify the type of reasoning: 'I have seen five dogs on my street, and all of them bark loudly. Therefore, all dogs bark loudly.'

2. Identify the type of reasoning: 'All squares are rectangles. ABCD is a square. Therefore, ABCD is a rectangle.'

3. Using the number pattern 5, 10, 15, 20, what is the conjecture for the next number?

$$5,\ 10,\ 15,\ 20,\ ?$$

4. A conclusion proved to be true by deductive reasoning is formally called what?

5. State whether the following conclusion is a conjecture or a theorem: 'Based on the pattern 2, 4, 6, 8, I predict the next number is 10.'

$$2,\ 4,\ 6,\ 8,\ ?$$

6. Using the Law of Detachment, given 'If it rains, then the ground gets wet' is true and 'It is raining' is true, what is the valid conclusion?

$$p \Rightarrow q,\ p \text{ is true}$$

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7. Using the Law of Detachment, determine if the following is a valid conclusion: 'If a student scores above 90, then the student gets an A. Maria scored 95. Therefore, Maria gets an A.'

$$p \Rightarrow q,\ p \text{ is true} \therefore q$$

8. Determine whether the following uses inductive or deductive reasoning: 'If all multiples of 4 are even, and 28 is a multiple of 4, then 28 is even.'

$$4k = 28,\ k = 7$$

9. Using the Law of Detachment, is the following argument valid? 'If two angles are supplementary, then their measures add up to 180°. Angles X and Y are supplementary. Therefore, m∠X + m∠Y = 190°.'

$$m\angle X + m\angle Y = 190^\circ$$

10. Write a full logical argument using the Law of Detachment with the true conditional statement 'If a polygon has exactly three sides, then it is a triangle,' and apply it to the figure with vertices A, B, and C connected by exactly three sides.

$$p \Rightarrow q:\ 3 \text{ sides} \Rightarrow \text{triangle}$$

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Deductive vs. Inductive Reasoning & The Law of Detachment — Answer Key

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Answer Key

1. Answer: Inductive reasoning
- The conclusion is based on observing a pattern from several specific examples.
- Since the conclusion is drawn from examples and may not always be true, this is inductive reasoning.

2. Answer: Deductive reasoning
- The conclusion is derived from a known general statement (all squares are rectangles).
- Because a specific conclusion is proved from a general true statement, this is deductive reasoning.

3. Answer: 25
- Observe that each term increases by 5 (multiples of 5).
- Following the pattern, the next number is 20 + 5 = 25. This is a conjecture formed by inductive reasoning.

4. Answer: A theorem
- By definition, deductive reasoning proves a specific conclusion from general statements.
- When that conclusion is proven true, it is called a theorem.

5. Answer: Conjecture
- The conclusion is based on observing a pattern, which is inductive reasoning.
- A conclusion from inductive reasoning is called a conjecture because it may or may not always be true.

6. Answer: The ground gets wet.
- Identify p: 'It is raining' and q: 'The ground gets wet.'
- Since the conditional $p \rightarrow q$ is true and p is true, by the Law of Detachment, q must also be true.

7. Answer: Valid conclusion
- p: 'Maria scored above 90' is true (95 > 90). q: 'Maria gets an A.'
- Since $p \rightarrow q$ is assumed true and p is true, the Law of Detachment confirms q is true — the conclusion is valid.

8. Answer: Deductive reasoning
- A general true statement ('all multiples of 4 are even') is used as the basis.
- A specific conclusion about 28 is drawn from this general rule, making it deductive reasoning.

9. Answer: Not valid (invalid conclusion)
- The conditional states: if supplementary, then sum = 180°. Angles X and Y are supplementary, so q should be $m\angle X + m\angle Y = 180°$.
- The stated conclusion of 190° contradicts the conditional statement, so the argument is not a valid application of the Law of Detachment.

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10. Answer: Triangle ABC is a triangle.

- Identify the conditional: p = 'a polygon has exactly three sides,' q = 'it is a triangle.' This is assumed true.
- Identify that figure ABC has exactly three sides, so p is true.
- By the Law of Detachment, since p → q is true and p is true, conclude q is true: Triangle ABC is a triangle — a valid deductive conclusion (theorem).

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