Biconditional Statements in Geometry

Geometry Worksheet · Grade 9–10

Name: _____ Date: _____

Learning Objectives

- Identify and write biconditional statements using 'if and only if'
- Decompose a biconditional statement into its conditional and converse
- Determine the truth value of a biconditional statement based on its conditional and converse

Problems

1. Which phrase is used to form a biconditional statement?

2. Identify the symbol used to represent a biconditional statement in mathematics.

$p \leftrightarrow q$

3. Identify the two parts (p and q) of the following biconditional statement: 'Two angles are supplementary if and only if the sum of their measures is 180 degrees.'

$p \leftrightarrow q$

4. Write the conditional statement (p $\rightarrow$ q) from the biconditional: 'A closed figure is a triangle if and only if it has three sides.'

$p \rightarrow q$

5. Write the converse (q $\rightarrow$ p) of the biconditional: 'A closed figure is a triangle if and only if it has three sides.'

$q \rightarrow p$

6. Determine whether the following biconditional statement is true or false: 'Two angles are supplementary if and only if the sum of their measures is 180 degrees.'

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$p \leftrightarrow q$

7. Rewrite the following biconditional in symbolic form: 'An angle is a right angle if and only if its measure is 90 degrees.'

$p \leftrightarrow q$

8. A student writes this biconditional: 'A number is even if and only if it ends in 1.' Determine if this is a true biconditional and explain why.

$p \leftrightarrow q$

9. Given p: 'Lines are parallel' and q: 'Lines do not intersect,' write the full biconditional statement and its conditional and converse, then evaluate the truth of the biconditional.

$p \leftrightarrow q$

10. Write a valid biconditional statement about complementary angles, decompose it into its conditional and converse, verify both are true, and express it in symbolic notation.

$p \leftrightarrow q$

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Biconditional Statements in Geometry — Answer Key

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Answer Key

1. Answer: If and only if
- A biconditional statement connects two parts using a special verbal operation.
- The phrase 'if and only if' (abbreviated 'iff') is used to form a biconditional statement.

2. Answer: Double-headed arrow (↔)
- Conditional statements use a single arrow: p \rightarrow q.
- Biconditional statements use a two-sided (double-headed) arrow: p \leftrightarrow q.

3. Answer: p: Two angles are supplementary; q: The sum of their measures is 180 degrees
- The part before 'if and only if' is p: 'Two angles are supplementary.'
- The part after 'if and only if' is q: 'The sum of their measures is 180 degrees.'

4. Answer: If a closed figure is a triangle, then it has three sides.
- Identify p: 'A closed figure is a triangle' and q: 'it has three sides.'
- Write the conditional as: If p, then q → 'If a closed figure is a triangle, then it has three sides.'

5. Answer: If a closed figure has three sides, then it is a triangle.
- Identify q: 'it has three sides' and p: 'a closed figure is a triangle.'
- Switch p and q to form the converse: 'If a closed figure has three sides, then it is a triangle.'

6. Answer: True
- Check the conditional: If two angles are supplementary, then their measures sum to 180°. This is TRUE by definition.
- Check the converse: If the sum of two angle measures is 180°, then they are supplementary. This is also TRUE. Since both are true, the biconditional is TRUE.

7. Answer: p ↔ q, where p: An angle is a right angle; q: Its measure is 90 degrees
- Let p = 'An angle is a right angle' and q = 'Its measure is 90 degrees.'
- Write in symbolic form: p \leftrightarrow q.

8. Answer: False — the converse is false
- Conditional: If a number is even, then it ends in 1. This is FALSE (e.g., 2 is even but does not end in 1).
- Since the conditional is false, the biconditional is FALSE. A biconditional is true only when BOTH the conditional and its converse are true.

9. Answer: True biconditional: 'Lines are parallel if and only if they do not intersect.'
- Conditional (p → q): 'If lines are parallel, then they do not intersect.' TRUE.

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• Converse (q → p): 'If lines do not intersect, then they are parallel.' TRUE (in a plane). Since both are true, the biconditional p \leftrightarrow q is TRUE.

10. Answer: p ↔ q is TRUE; 'Two angles are complementary if and only if the sum of their measures is 90 degrees.'

• Let p: 'Two angles are complementary' and q: 'The sum of their measures is 90 degrees.' Biconditional: 'Two angles are complementary if and only if the sum of their measures is 90 degrees.'

• Conditional (p → q): 'If two angles are complementary, then their measures sum to 90°.' TRUE. Converse (q → p): 'If the sum of two angle measures is 90°, then the angles are complementary.' TRUE. Therefore p \leftrightarrow q is TRUE.

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